CHEMISTRY 163A FALL 2002

PROBLEM SET

1. A simple differential equation which is important to quantum mechanics is

$$\frac{df(t)}{dt} = \frac{-i}{\hbar} E \ f(t)$$

where E, $\hbar$, and i [i = (- 1)^{½}] are constants. Solve this differential equation for f(t) with the boundary condition f(0) = 1.

2. In this problem we demonstrate that classical mechanics and classical electrodynamics would predict the rapid collapse of the hydrogen atom in the Rutherford model.

   (a) From classical electrodynamics, the intensity of radiation given off by an electron circulating within an angular velocity ω at a distance r from the nucleus us:

$$I = \frac{dE_{rad}}{dt} = -\frac{dE_H}{dt} = \frac{e^2 \omega^4 r^2}{6\pi\varepsilon_o c^3}$$

   where E_{rad} is the energy of the emitted radiation and E_H is the energy of the hydrogen atom. Use the chain rule to find an expression for $\frac{dr}{dt}$ in terms of $\frac{dE_{rad}}{dt}$ and $\frac{dr}{dE_H}$.

   (b) Using the result from (a) and relationships derived in class for the Rutherford hydrogen atom, show that $\frac{dr}{dt} = \frac{-4}{3Zc^3}\left(\frac{2Ze^2}{8\pi\varepsilon_0 rm}\right)^2$

   (c) Solving this differential equation, calculate the time it would take for an electron initially at the Bohr radius, $a_0 = 5.29 \times 10^{-11}$ m to collapse into the nucleus (you should look up a reasonable estimate for the radius of the nucleus).

3. Problem #1- 23 McQ (parts a and b) and

   c) Calculate the De Broglie wavelength of an Edsel weighing 900 kg (~ 1 ton) and traveling 0.027 km/sec (~60 mph). What is the implication of the relative magnitudes of the wavelengths calculated in parts a, b, and c?

4. In lecture we stated that the relativistic energy of a free electron was

$$E = (m_0^2 c^4 + p^2 c^2)^{1/2}$$

where

$$p = \frac{m_0 v}{(1 - v^2/c^2)^{1/2}}$$

is the relativistic expression for momentum.

a) Show that in the correspondence limit, v << c, the above expression becomes

$$E = m_0 c^2 + 1/2\, m_0 v^2$$

b) What is the significance of each of the terms in the above expression?

5. A mass (m) attached to a spring is one, idealized, example of a harmonic oscillator. Here the mass is acted upon by the form $F = -kx$, where $x = 0$ is the resting (equilibrium) position of the spring and k and the spring constant.

a) Using Newton's law, $F = ma$, write a differential equation for the motion of a classical harmonic oscillator.

b) If the spring is fully extended to a length A at time t = 0, what is

$$\left(\frac{dx}{dt}\right)_{t=0} = ?$$

c) Show that $x(t) = A\cos\left[\left(\frac{k}{m}\right)^{\frac{1}{2}} t\right]$ satisfies the equation of motion and the boundary conditions in part (b).

d) What is the length τ of one period of vibration? Will a larger spring constant increase or decrease the period of oscillation? What effect will increasing the mass have on the period of vibration?

e) What is the average kinetic energy:

$$\overline{K.E.} = \frac{1}{\tau}\int_0^\tau \frac{1}{2} m[v(t)]^2\, dt$$

(i.e., evaluate)

*f) What is the average potential energy, assuming U(x=0) = 0 (you set up that integral and evaluate)?

*g) If you've gotten part (e) correct, there is a simple relationship between K.E. and P.E. for the harmonic oscillator. How could this have been obtained without direct evaluation of the integrals in part (e)?