

HYDROGEN ORBITALS WITH TWO ANGULAR NODES

{ $\ell = 2$, $|m| = 0, 1, 2$; d-orbitals, $(2\ell + 1) = 5$ components }

Cartesian functions with two nodal planes:

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|-------|------|------|-------------|---|
| (i) | xy | (iv) | $x^2 - y^2$ | but since (v) = (iv) + (vi)
set is <i>not</i> linearly independent |
| (ii) | xz | (v) | $z^2 - y^2$ | |
| (iii) | yz | (vi) | $z^2 - x^2$ | |

Choose 5 independent components:

$$xy, \quad xz, \quad yz, \quad x^2 - y^2, \quad 3z^2 - r^2$$

$$\quad \quad \quad \parallel$$

$$3z^2 - (x^2 + y^2 + z^2) = 2z^2 - x^2 - y^2 = (v) + (vi)$$

	Cartesian	Spherical Polar	$R(r)$	$Q(q)$	$F(f)$	ℓ	$ m $
a)	$(3z^2 - r^2)$	$r^2(3 \cos^2 q - 1)f(r)$	$r^2 f(r)$	$(3 \cos^2 - 1)$	1	2	0
b)	$xzf(r)$	$r^2(\cos q \sin q \cos f)f(r)$	$r^2 f(r)$	$(\cos q \sin q)$	$\cos f$	2	1
c)	$yzf(r)$	$r^2(\cos q \sin q \sin f)f(r)$	$r^2 f(r)$	$(\cos q \sin q)$	$\sin f$	2	1
d)	$xyf(r)$	$r^2(\sin q \cos f \sin q \sin f)f(r)$	$r^2 f(r)$	$\sin^2 q$	$\sin f \cos f$ $\parallel$ $1/2 \sin 2f$	2	2
e)	$(x^2 - y^2)f(r)$	$r^2(\sin^2 q \cos^2 f - \sin^2 q \sin^2 f)f(r)$	$r^2 f(r)$	$\sin^2 q$	$(\cos^2 f - \sin^2 f)$ $\parallel$ $1/2 \cos 2f$	2	2

You should be able to associate each $Q_{2|m|}(q)$ with the relevant series

from the “Math Review” as well as $F_m(f)$