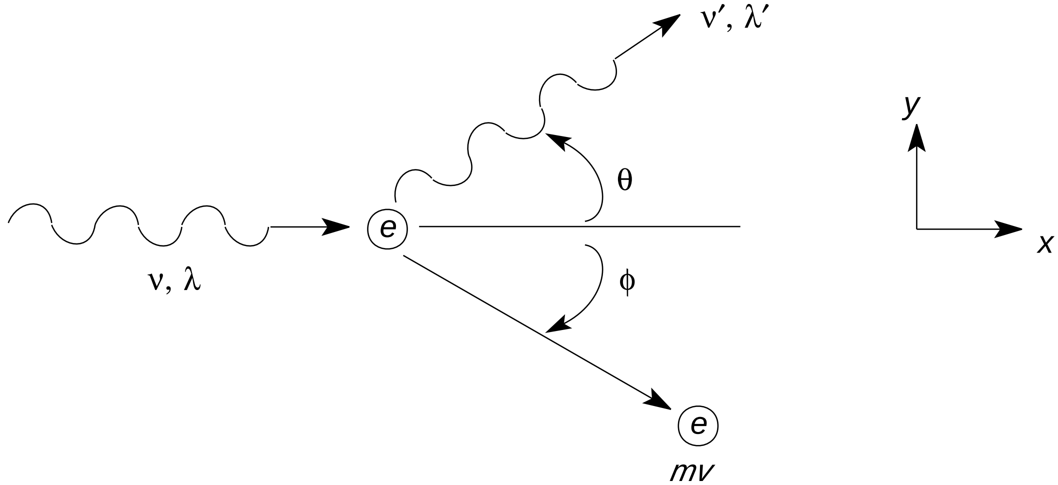
Chem 163A

COMPTON SCATTERING



RESULT: $\lambda' - \lambda = \frac{2h}{mc}\sin^2\left(\frac{\theta}{2}\right)$

STARTING RELATIONSHIPS

$$\text{relationships from Planck and De Broglie: } E = h\nu = \frac{hc}{\lambda} \qquad (1)$$

$$p\lambda = h \qquad (2)$$

$$\text{conservation of energy: } h\nu = h\nu' + \frac{mv^2}{2} \qquad (3)$$

$$\text{conservation of momentum (x-direction): } \frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + mv\cos\phi \qquad (4)$$

$$\text{conservation of momentum (y-direction): } 0 = \frac{h}{\lambda'}\sin\theta - mv\sin\phi \qquad (5)$$

ALGEBRA

from (3)

$$hc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)=\frac{mv^2}{2} \tag{6}$$

from (4)

$$mv\cos\phi=\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\theta \tag{7}$$

from (5)

$$mv\sin\phi=\frac{h}{\lambda'}\sin\theta \tag{8}$$

from ($7^2 + 8^2$)

$$m^2v^2(\cos^2\phi+\sin^2\phi)=\left(\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\theta\right)^2+\left(\frac{h}{\lambda'}\right)^2\sin^2\theta \tag{10}$$

using $\cos^2\phi + \sin^2\phi = 1$

$$m^2v^2=h^2\left[\left(\frac{1}{\lambda}\right)^2-\frac{2\cos\theta}{\lambda\lambda'}+\left(\frac{\cos\theta}{\lambda'}\right)+\left(\frac{\sin\theta}{\lambda'}\right)^2\right] \tag{11}$$

dividing by 2m

$$\frac{mv^2}{2}=\frac{h^2}{2m}\left[\left(\frac{1}{\lambda}\right)^2+\left(\frac{1}{\lambda'}\right)^2-\frac{2\cos\theta}{\lambda\lambda'}\right] \tag{12}$$

combining (6) and (12):

$$hc\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right)=\frac{h^2}{2m}\left[\left(\frac{1}{\lambda}\right)^2+\left(\frac{1}{\lambda'}\right)^2-\frac{2\cos\theta}{\lambda\lambda'}\right] \tag{13}$$

Assume a small shift, $\lambda \approx \lambda'$ (O.K. for terms involving addition or multiplication of λ, λ' but not for subtraction).

$$hc\left(\frac{\lambda'-\lambda}{\lambda^2}\right)=\frac{h^2}{2m}\left[\frac{2}{\lambda^2}-\frac{2\cos\theta}{\lambda^2}\right] \tag{14}$$

$$\lambda'-\lambda=\frac{h}{2mc}[2-2\cos\theta] \tag{15}$$

Use trig identity $(1-\cos\theta)=2\sin^2\left(\frac{\theta}{2}\right)$

$$\Delta\lambda=\lambda'-\lambda=\frac{2h}{mc}\sin^2\left(\frac{\theta}{2}\right) \qquad \textit{Q.E.D.}$$