MATHEMATICAL VOCABULARY FOR QUANTUM MECHANICS

Below are some concepts, terms, and examples of mathematical vocabulary relevant to our study of quantum mechanics.

1. DIFFERENTIAL EQUATIONS:

The equations which determine (as best as one can) the behavior of particles on a quantum mechanical basis are differential equations where the desired solution (a function) satisfies relationships involving derivatives. The fundamental differential equation of quantum mechanics is the Schrödinger time-dependent equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}+U(x)\psi(x,t)=\frac{i\hbar\partial\psi(x,t)}{\partial t}$$

(know: solution by simple integration, boundary conditions, substitution to prove solution)

2. OPERATORS

The concept of an operator is meaningful only in the terms of a rule which defines the operator's effect on a function:

operator (function 1) → function 2

[remember: function (number 1) → number 2]

Examples:

i) Differential operators

$$\left(\frac{d}{dx}\right)_{op} \quad f(x) \quad = \quad \frac{df}{dx}$$

↑ ↑ ↑

operator function result

$$\left(\frac{d}{dx}\right)_{op} x^2 = 2x$$

ii) Multiplicative operator (note: $x_{op} \equiv \hat{x}$ in the text’s notation)

$$x_{op} \quad f(x) \quad = \quad xf(x)$$

↑	↑	↑
operator	function #1	function #2 (result)

$$x_{op}[3x^3] = x3x^3 = 3x^4$$

3. PRODUCT OF OPERATORS

Examples

i) $$x_{op}\left(\frac{d}{dx}\right)_{op} f(x) = x_{op}\frac{df}{dx} = \frac{xdf}{dx}$$

ii) $$\left[\left(\frac{d}{dx}\right)_{op} x_{op}\right] f(x) = \left(\frac{d}{dx}\right)_{op} xf(x)$$

$$= f(x) + x\frac{df}{dx}$$

Note: from examples ii) and iii) the results depend on the order of the operators.

iii) $$\left(\frac{d}{dx}\right)_{op}\left(\frac{d}{dx}\right)_{op} f(x) = \left(\frac{d^2}{dx^2}\right)_{op} f(x) = \frac{d^2 f}{dx^2}$$

4. EIGENVALUES OF AN OPERATOR (note $A_{op} \equiv \hat{A}$ in the text's notation)

$A_{op}\, f(x) \quad = \quad a\ f(x)$ for <u>some(s)</u> *f(x)*

↑ operator ↑ const(s).

f(x) is an <u>eigenfunction</u> of the operator A_{op} with <u>eigenvalue</u> *a*.

<u>Example</u>

$$\text{for } A_{op} = \left(\frac{d^2}{dx^2}\right)_{op} \text{ and } f(x) = \sin kx$$

$$\left(\frac{d^2}{dx^2}\right)_{op} \sin kx = -k^2 \sin kx$$

<u>Yes</u> sin kx is an <u>eigenfunction</u> of

$$\left(\frac{d^2}{dx^2}\right)_{op} \text{ with eigenvalue } -k^2.$$

5. **COMMUTATION** (section 4-8 McQ)

The commutator (or commutation bracket) for operator $\hat{A}$ and $\hat{B}$ the commutator is given by $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$. If the action of the commutator on any well-behaved function gives a specific result (value), the commutator is assigned this value. For example we will show

$$[\hat{x}, \hat{p}_x] f(x) = f(x) = (\hat{x}\hat{p}_x - \hat{p}_x\hat{x}) f(x) = i\hbar f(x)$$

$$\text{thus } [\hat{x}, \hat{p}_x] = i\hbar\hat{I}.$$