Outline of Solution of Quantum Mechanical Harmonic Oscillator *(Eugene Switkes, Univ of Calif Santa Cruz)*

Schrödinger time dependent equation

$$\hat{H}\Psi(x_1, x_2, t) = -\frac{\hbar}{i}\frac{\partial \Psi}{\partial t}$$

$$f(t) = e^{-\frac{i}{\hbar}Et}$$

Schrödinger time independent equation

$$\hat{H}\psi'(x_1, x_2) = E\,\psi'(x_1, x_2)$$

Center − of − mass and relative (vibrational) motion

$$\psi'(x_1, x_2) = \psi_{cm}(x_{cm})\psi_r(x_r)$$

$$\psi_{cm}$$

$$\lambda = \frac{2\mu E}{\hbar^2} \qquad \alpha = \frac{2\pi\mu\upsilon_0}{\hbar}$$

$$\xi = \alpha^{\frac{1}{2}}x$$

MacQ, sec 5-11

from asymptotic behavior

$$\psi_r = Be^{-\frac{\xi^2}{2}}H(\xi)$$

MacQ, sec 5-11

equation for $H(\xi)$

MacQ, sec 5-11, eqn (5-76)

$$H(\xi) = \sum_k a_k \xi^k$$

MacQ, sec 5-12

collect coefficients of ξ^k *and set equal to 0 for each* k

MacQ, sec 5-12, eqns (5-78), (5-79)

recursion relationship

$$a_{k+2} = -\frac{\left(\frac{\lambda}{\alpha} - 2k - 1\right)}{(k+2)(k+1)}a_k$$

MacQ, sec 5-12, eqn (5-80)

either all $a_{k_{even}} = 0$ *or all* $a_{k_{odd}} = 0$ *and terminate remaining series to satisfy*

$$\lim_{x\to\infty} \psi(x) = 0$$

MacQ, sec 5-12

$$\lambda_n = (2n+1)\alpha$$

$$E_n = \left(n + \frac{1}{2}\right)h\upsilon_0$$

$$\psi_n = B_n e^{-\frac{\xi^2}{2}}H_n(\xi)$$

$$H_n(\xi) = a_0 + a_2\xi^2 \ldots + a_n\xi^n \qquad or \qquad H_n(\xi) = a_1\xi + a_3\xi^3 \ldots + a_n\xi^n$$