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Chemistry 163A

Indeterminacy Principle,
"Which slit??"
Measurement interacts with system!!

from: *Quantum Chemistry,*
by John P. Lowe,
Academic Press, New York, 1978,
pp. 16-19.

1-8 A Diffraction Experiment with Electrons

In order to gain a better understanding of the meaning of matter waves, we now consider a set of simple experiments. Suppose that we have a source of a beam of monoenergetic electrons and a pair of slits, as indicated schematically in Fig. 1-10. Any electron arriving at the phosphorescent screen produces a flash of light, just as in a television set. For the moment we ignore the light source near the slits (assume that it is turned off) and inquire as to the nature of the image on the phosphorescent screen when the electron beam is directed at the slits. The observation, consistent with the observations of Davisson and

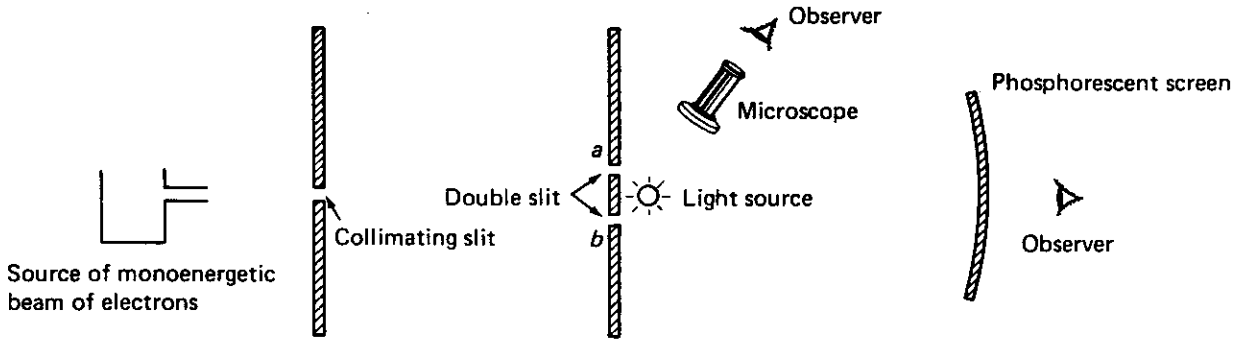

FIG. 1-10 The electron source produces a beam of electrons, some of which pass through slits *a* and/or *b* to be detected as flashes of light on the phosphorescent screen.

Germer already mentioned, is that there are alternating bands of light and dark, indicating that the electron beam is being diffracted by the slits. Furthermore, the distance separating the bands is consistent with the de Broglie wavelength corresponding to the energy of the electrons. The variation in light intensity observed on the screen is depicted in Fig. 1-11a.

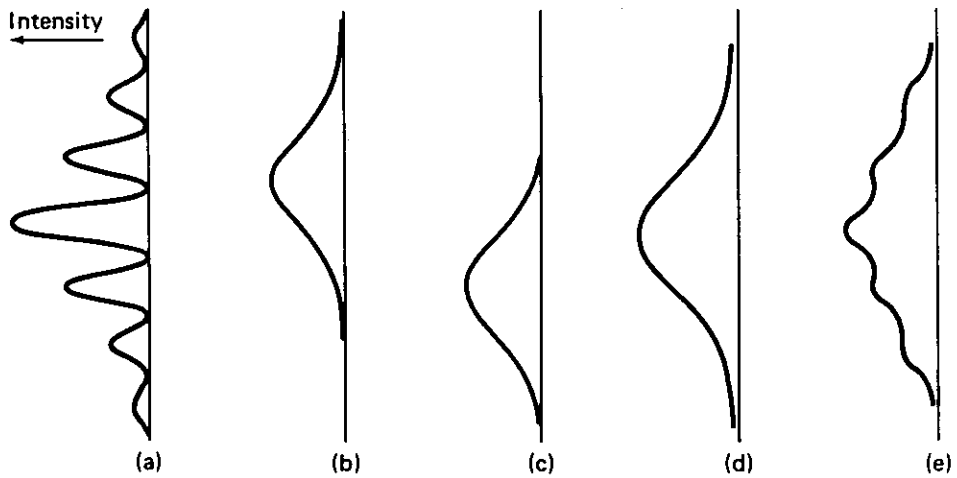

FIG. 1-11 Light intensity at phosphorescent screen under various conditions: (a) *a* and *b* open, light off; (b) *a* open, *b* closed, light off; (c) *a* closed, *b* open, light off; (d) *a* and *b* open, light on, λ short; (e) *a* and *b* open, light on, λ longer.

Evidently, the electrons in this experiment are displaying wave behavior. Does this mean that the electrons are spread out like waves when they are detected at the screen? We test this by reducing our beam intensity to let only one electron per second through the apparatus and observe that each electron gives a localized pinpoint of light, the entire diffraction pattern building up gradually by the accumulation of many points. Thus, the square of de Broglie's matter wave has the same kind of statistical significance that Einstein proposed for electromagnetic waves and photons, and the electrons really are localized particles, at least when they are detected at the screen.

However, if they are really particles, it is hard to see how they can be diffracted. Consider what happens when slit *b* is closed. Then all the electrons striking the screen must have come through slit *a*. We observe the result to be a single area of light on the screen (Fig. 1-11b). Closing slit *a* and opening *b* gives a similar (but displaced) light area, as shown in Fig. 1-11c. These patterns are just what we would expect for particles. Now, with both slits open, we expect half the particles to pass through slit *a* and half through slit *b*, the resulting pattern being the *sum* of the results just described. Instead we obtain the diffraction pattern (Fig. 1-11a). How can this happen? It seems that, somehow, an electron passing through the apparatus can sense whether one or both slits are open, even though as a particle it can explore only one slit or the other. One might suppose that we are seeing the result of simultaneous traversal of the two slits by two electrons, the path of each electron being affected by the presence of an electron in the other slit. This would explain how an electron passing through slit *a* would "know" whether slit *b* was open or closed. But the fact that the pattern builds up even when electrons pass through at the rate of one per second indicates that this argument will not do. Could an electron be coming through both slits at once?

To test this question, we need to have detailed information about the positions of the electrons as they pass through the slits. We can get such data by turning on the light source and aiming a microscope at the slits. Then photons will bounce off each electron as it passes the slits and will be observed through the microscope. The observer thus can tell through which slit each electron has passed, and also record its final position on the phosphorescent screen. In this experiment, it is necessary to use light having a wavelength short in comparison to the interslit distance; otherwise the microscope cannot resolve a flash well enough to tell which slit it is nearest. When this experiment is performed, we indeed detect each electron as coming through one slit or the other, and not both, but we also find that the diffraction pattern on the screen has been lost and that we have the broad, featureless distribution shown in Fig. 1-11d, which is basically the sum of the single-slit experiments. What has happened is that the photons from our light source, in bouncing off the electrons as they emerge from the slits, have affected the momenta of the electrons and changed their paths from what they were in the absence of light. We can try to counteract this by using photons with lower momentum; but this means using photons of lower E, hence longer λ. As a result, the images of the electrons in the microscope get broader, and it becomes more and more ambiguous as to which slit a given electron has passed through or that it really passed through only one slit. As we become more and more uncertain about the path of each electron as it moves past the slits, the accumulating diffraction pattern becomes more and more pronounced (Fig. 1-11e). (Since this is a "thought experiment," we can ignore the inconvenient fact that our "light" source must produce X rays or γ rays in order to have a wavelength short in comparison to the appropriate interslit

This conceptual experiment illustrates a basic feature of microscopic systems—we cannot measure properties of the system without affecting the future development of the system in a nontrivial way. The system with the light turned off is significantly different from the system with the light turned on (with short λ), and so the electrons arrive at the screen with different distributions. No matter how cleverly one devises his experiment, there is some minimum necessary disturbance involved in any measurement. In this example with the light off, the problem is that we know the momentum of each electron quite accurately (since the beam is monoenergetic and collimated), but we do not know anything about the way the electrons traverse the slits. With the light on, we obtain information about electron position just beyond the slits but we change the momentum of each electron in an unknown way. The measurement of particle position leads to loss of knowledge about particle momentum. This is an example of the *uncertainty principle* of Heisenberg, who stated that the product of the simultaneous uncertainties in "conjugate variables," a and b, can never be smaller than a number on the order of Planck's constant h:

$$\Delta a \cdot \Delta b \gtrsim h = 6.626176 \times 10^{-34} \quad \text{J sec} \tag{1-46}$$

Here, Δa is a measure of the uncertainty in the variable a, etc. (The easiest way to recognize conjugate variables is to note that their dimensions must multiply to joule seconds. Linear momentum and linear position satisfies this requirement. Two other important pairs of conjugate variables are energy–time and angular momentum–angular position.) In this example with the light off, our uncertainty in momentum is small and our uncertainty in position is unacceptably large, since we cannot say which slit each electron traverses. With the light on, we reduce our uncertainty in position to an acceptable size, but subsequent to the position of each electron being observed, we have much greater uncertainty in momentum.

Thus, we see that the appearance of an electron (or a photon) as a particle or a wave depends on our experiment. Because *any* observation on so small a particle involves a significant perturbation of its state, it is proper to think of the electron plus apparatus as a single system. The question, "Is the electron a particle or a wave?" becomes meaningful only when the apparatus is defined on which we plan a measurement. In some experiments, the apparatus and electrons interact in a way suggestive of the electron being a wave, in others, a particle. The question, "What is the electron when we're not looking?," cannot be answered experimentally, since an experiment *is* a "look" at the electron.