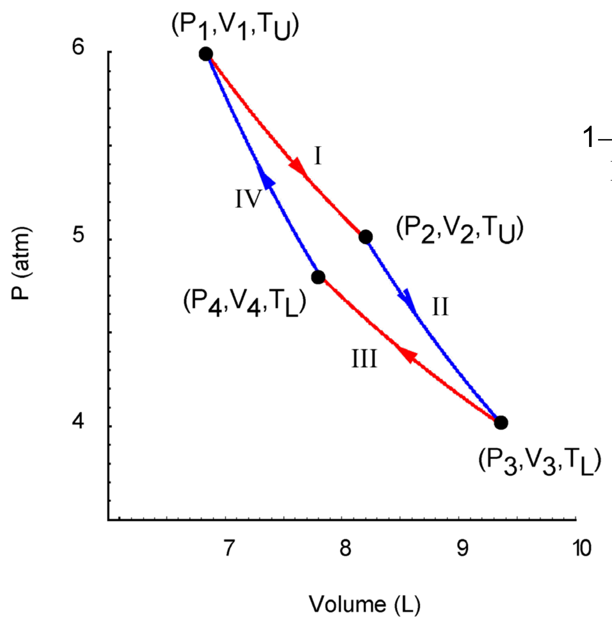
Summary of Heat and Work for the Carnot Cycle
Engines, Refrigerators, Heat Pumps

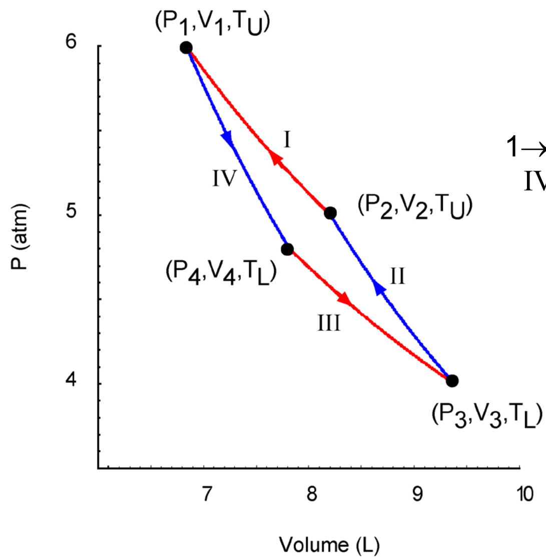
Engine (heat into system, net work out), forward cycle
(see Carnot Arithmetic Review Handout for details of each step; relevant equation numbers indicated):



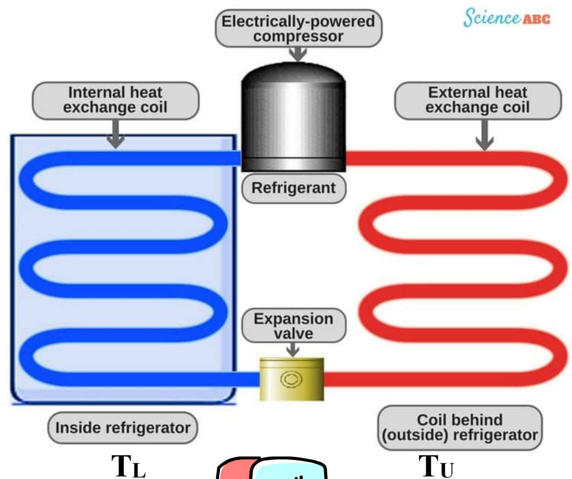
1→2→3→4→1
I II III IV

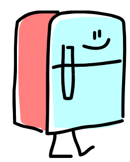
ENGINE	q	w_{sys}	w_{surr}	
I. isothermal expansion	$+nRT_U \ln\frac{P_1}{P_2}$ **1.3**	$-nRT_U \ln\frac{P_1}{P_2}$ **1.2**	$+nRT_U \ln\frac{P_1}{P_2}$	**heat in at T_H** **work out**
II adiabatic expansion	0	$n\overline{C}_V(T_L - T_U)$ **2.4**	$-n\overline{C}_V(T_L - T_U)$	**work out**
III. isothermal compression	$nRT_L \ln\frac{P_3}{P_4} = -nRT_L \ln\frac{P_1}{P_2}$ **3.3&T.3**	$-nRT_L \ln\frac{P_3}{P_4} = nRT_L \ln\frac{P_1}{P_2}$ **3.2&T.3**	$-nRT_L \ln\frac{P_1}{P_2}$	heat lost at T_L **work in**
IV. adiabatic compression	0	$n\overline{C}_V(T_U - T_L)$ **4.4**	$-n\overline{C}_V(T_U - T_L)$	**work in**
net gain/cost	$q_{in} = q_I = q_U$ $+nRT_U \ln\frac{P_1}{P_2}$	$(w_{total})_{sys}$= $(w_I+w_{II}+w_{III}+w_{IV})_{sys}$= $-nR(T_U - T_L)\ln\frac{P_1}{P_2}$	$(w_{total})_{surr}$= $(w_I+w_{II}+w_{III}+w_{IV})_{surr}$= - $(w_{total})_{sys}$ $nR(T_U - T_L)\ln\frac{P_1}{P_2}$	$\varepsilon = w_{surr}/q_{in}$ $\varepsilon = (T_U - T_L)/T_U$

Refrigerator is 'reverse' Carnot cycle; here the system is the refrigerator motor, heat coils, and room and the surroundings the refrigerator (in contact with the internal heat exchange coils). The quantity of interest is the heat removed from the surroundings (the ice box) by isothermal expansion at T_L and the cost is the work done on system *[this definition of system and surroundings differs from that of E&R$_{4th}$ p 136;however I believe that this definition is more intuitive; the resulting formula for efficiency is identical]* .
(see Carnot Arithmetic review handout for details of each step



1→4→3→2→1
IV III II I





https://www.scienceabc.com/innovation/how-does-a-refrigerator-work-working-principle.html

REFRIGERATOR	q	w_{sys}	q_{surr}	
IV. adiabatic expansion $T_H \rightarrow T_L$	0	$-n\overline{C_V}(T_U - T_L)$	0	**work done by system**
III. isothermal expansion at T_L $P_4 \rightarrow P_3$	$-nRT_L \ln\frac{P_3}{P_4} = nRT_L \ln\frac{P_1}{P_2}$	$+nRT_L \ln\frac{P_3}{P_4} = -nRT_L \ln\frac{P_1}{P_2}$	$-nRT_L \ln\frac{P_1}{P_2}$	**work done by sys** **heat withdrawn from T_L (ice box)**
II adiabatic compression $T_L \rightarrow T_H$	0	$-n\overline{C_V}(T_L - T_U)$	0	**work on system**
I. isothermal compression at T_H $P_2 \rightarrow P_1$	$-nRT_U \ln\frac{P_1}{P_2}$	$+nRT_U \ln\frac{P_1}{P_2}$	$+nRT_U \ln\frac{P_1}{P_2}$	heat out at T_H refrig "coils" **work on system**
net gain/cost		$w_{total}=w_I+w_{II}+w_{III}+w_{IV}=$ $+nR(T_U - T_L)\ln\frac{P_1}{P_2}$	$\lvert q_{out}\rvert = -\, q_{surr\_III}$ $+nRT_L \ln\frac{P_1}{P_2}$	$\eta_r \equiv C_R = \frac{-q_{surr\_III}}{w_{sys\ total}} = \frac{T_L}{T_U - T_L}$ eqn 5.69 E&R$_{4th}$ *[5.45 E&R$_{3rd}$]*

Heat Pump (reverse Carnot like refrigerator, but the measure of performance depends on the heat put into the room at T_H , i.e. step I)

$$\eta_{hp} \equiv C_{HP} = \frac{q_{surr\_I}}{w_{total}} = \frac{T_U}{T_U - T_L} > 1$$

eqn 5.68 E&R$_{4th}$ *5.44 [E&R$_{3rd}$]*