Reversible Transfer of heat from 'block 1' at T_H to 'block 2' T_L
(two blocks have same heat capacity C_B, i.e. same mass and same specific heat)

      In problem #29 we calculated the entropy change when two isolated blocks of iron at differing temperatures are brought together and an **irreversible, adiabatic,** heat exchange occurs. For heating/cooling we apply the same formulas for calculating ΔS of a material. It may seem strange that one would get, for the blocks of iron, the exact same ΔS as for the **reversible** and **irreversible** heating/cooling of the blocks. What then would be different in the irreversible and reversible cases?? The intent of this handout is to present how one would accomplish the heat transfer **reversibly** and show that for the reversible and irreversible cases the ΔS is indeed the same for the blocks of iron. However the reversible path between the same initial and final states of the system (blocks) is not adiabatic. Calculating q_{rev} for this reversible path does give $\Delta S = \int d_{qrev}/T$

      The following hold (for the entropy changes when two 'blocks' of differing temperatures exchange heat):

$$\Delta S_{blocks} = C_B \left[\ln \frac{T_F}{T_H} + \ln \frac{T_F}{T_L} \right] = C_B \left[\ln \frac{T_F{}^2}{T_H T_L} \right] = C_B \left[\ln \frac{(T_H + T_L)^2}{4 T_H T_L} \right]$$

    which is true for both reversible and irreversible heat transfers

for isolated, irreversible, heat exchange:

$$\Delta S_{surr} = 0 \quad and \quad (\Delta S_{total})_{irrev} = \Delta S_{blocks} > 0$$

but for reversible heat exchange (as calculated below):

$$\Delta S_{surr} = -C_B \left[\ln \frac{T_F{}^2}{T_H T_L} \right] \quad and \quad (\Delta S_{total})_{rev} = \Delta S_{surr} + \Delta S_{blocks} = 0$$

In the reversible process, the blocks are heated/cooled by reversible expansions and compressions of an ideal gas that are adiabatic except for heat exchange with the block (consider the ideal gas + block to be the 'system'). The process described will be a reversible process bringing block at T_L and block at T_H each to T_F and leave the gas and the piston unchanged at the end of the cycle. The following (0.1) applies to the heating/cooling of the blocks by the pseudo 'adiabatic' compression/expansions.

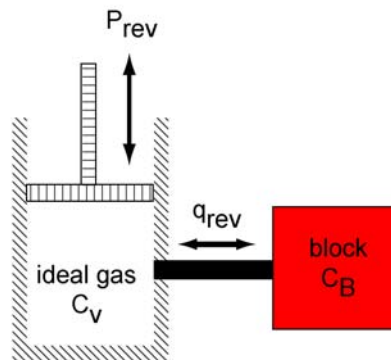$$dU = C_V dT = đq - PdV$$

$$đq = -C_B dT$$

$$P = \frac{RT}{V} \qquad\qquad (0.1)$$

$$C_V dT = -C_B dT - \frac{RT}{V} dV$$

$$\frac{C_V + C_B}{RT} dT = -\frac{dV}{V}$$

$$\ln \left(T^{\frac{C_V + C_B}{R}} \right) = -\ln(V)$$

$$V T^{\frac{C_V + C_B}{R}} = const$$

Step I: Reversible $T_H \rightarrow T_F$, absorb heat from block 1. (V_1, P_1, $T_H \rightarrow V_2$, P_2, T_F)

$$V_2 T_F^{\frac{C_V+C_B}{R}} = V_1 T_H^{\frac{C_V+C_B}{R}}$$

$$\Delta S_{1 \rightarrow 2} = C_V \ln\left(\frac{T_F}{T_H}\right) + R \ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S_{1 \rightarrow 2} = C_V \ln\left(\frac{T_F}{T_H}\right) + R \ln\left(\frac{T_H^{\frac{C_V+C_B}{R}}}{T_F^{\frac{C_V+C_B}{R}}}\right) \qquad\qquad \text{(I.1)}$$

$$\Delta S_{1 \rightarrow 2} = C_V \ln\left(\frac{T_F}{T_H}\right) - R\left(\frac{C_V+C_B}{R}\right)\ln\left(\frac{T_F}{T_H}\right)$$

$$\Delta S_{1 \rightarrow 2} = C_B \ln\left(\frac{T_H}{T_F}\right)$$

Step II: Reversible adiabatic expansion $T_F \rightarrow T_L$ of ideal gas only:
(V_2, P_2, $T_F \rightarrow V_3$, P_3, T_L) :

$$V_2 T_F^{\frac{C_V}{R}} = V_3 T_L^{\frac{C_V}{R}} \qquad\qquad \text{(II.1)}$$
$$\Delta S_{2 \rightarrow 3} = 0$$

Step III: Reversible $T_L \rightarrow T_F$, transfer heat to block 2 . (V_3, P_3, $T_L \rightarrow V_4$, P_4, T_F)

$$V_3 T_L^{\frac{C_V+C_B}{R}} = V_4 T_F^{\frac{C_V+C_B}{R}}$$

$$\Delta S_{3 \rightarrow 4} = C_V \ln\left(\frac{T_F}{T_L}\right) + R \ln\left(\frac{V_4}{V_3}\right)$$

$$\Delta S_{3 \rightarrow 4} = C_V \ln\left(\frac{T_F}{T_L}\right) + R \ln\left(\frac{T_L^{\frac{C_V+C_B}{R}}}{T_F^{\frac{C_V+C_B}{R}}}\right) \qquad\qquad \text{(III.1)}$$

$$\Delta S_{3 \rightarrow 4} = C_V \ln\left(\frac{T_F}{T_L}\right) - R\left(\frac{C_V+C_B}{R}\right)\ln\left(\frac{T_F}{T_L}\right)$$

$$\Delta S_{3 \rightarrow 4} = C_B \ln\left(\frac{T_L}{T_F}\right)$$

To return gas to initial state:

Step IV: Reversible adiabatic compression $T_F \rightarrow T_H$ of ideal gas only:
(V_4, P_4, $T_F \rightarrow V_5$, P_5, T_H) :

$$V_4 T_F^{\frac{C_V}{R}} = V_5 T_H^{\frac{C_V}{R}} \qquad \text{(IV.1)}$$
$$\Delta S_{4 \rightarrow 5} = 0$$

Will $V_5 = V_1$?

$$V_5 = V_4 \left(\frac{T_F}{T_H}\right)^{\frac{C_V}{R}}$$

$$= V_3 \left(\frac{T_L}{T_F}\right)^{\frac{C_V + C_B}{R}} \left(\frac{T_F}{T_H}\right)^{\frac{C_V}{R}} V_2 \left(\frac{T_F}{T_L}\right)^{\frac{C_V}{R}} \left(\frac{T_L}{T_F}\right)^{\frac{C_V + C_B}{R}} \left(\frac{T_F}{T_H}\right)^{\frac{C_V}{R}}$$

$$= V_1 \left(\frac{T_H}{T_F}\right)^{\frac{C_V + C_B}{R}} \left(\frac{T_F}{T_L}\right)^{\frac{C_V}{R}} \left(\frac{T_L}{T_F}\right)^{\frac{C_V + C_B}{R}} \left(\frac{T_F}{T_H}\right)^{\frac{C_V}{R}}$$

$$= V_1 \left(\frac{T_H T_L}{T_F^{\,2}}\right)^{\frac{C_V + C_B}{R}} \left(\frac{T_F^{\,2}}{T_H T_L}\right)^{\frac{C_V}{R}}$$

$$= V_1 \left(\frac{T_H T_L}{T_F^{\,2}}\right)^{\frac{C_B}{R}}$$

(IV.1)

No, so reversible isothermal volume change (V_5, P_5, $T_H \rightarrow V_1$, P_1, T_H) needs Step V:

$$\Delta S_{5\rightarrow 1} = R \ln \frac{V_1}{V_5} = R \ln \left(\frac{T_F^{\,2}}{T_H T_L}\right)^{\frac{C_B}{R}}$$

$$= C_B \ln\left(\frac{T_F}{T_H}\right) + C_B \ln\left(\frac{T_F}{T_L}\right)$$

(V.1)

note that $\dfrac{T_F^{\,2}}{T_H T_L} > 1$ (heat absorbed):

$$\left(\frac{T_H + T_L}{2}\right)^2 > 0 \Rightarrow \frac{1}{4}\left(T_H^{\,2} + T_L^{\,2} + 2T_H T_L\right) > 0$$

$$\left(T_H^{\,2} + T_L^{\,2}\right) > 2T_H T_L$$

$$\frac{T_F^{\,2}}{T_H T_L} = \frac{\frac{1}{4}\left(T_H^{\,2} + T_L^{\,2} + 2T_H T_L\right)}{T_H T_L} > \frac{\frac{1}{4}\left(2T_H T + 2T_H T_L\right)}{T_H T_L} = 1$$

Total entropy for reversible process of ideal gas $1\rightarrow 2\rightarrow 3\rightarrow 4\rightarrow 5\rightarrow 1$:

$$\Delta S_{total\ for\ gas} = \Delta S_{1\rightarrow 2} + \Delta S_{2\rightarrow 3} + \Delta S_{3\rightarrow 4} + \Delta S_{4\rightarrow 5} + \Delta S_{5\rightarrow 1}$$

$$= C_B \ln\left(\frac{T_H}{T_F}\right) + 0 + C_B \ln\left(\frac{T_L}{T_F}\right) + 0 + C_B \ln\left(\frac{T_F}{T_H}\right) + C_B \ln\left(\frac{T_F}{T_L}\right) \qquad \text{TOTAL FOR GAS}$$

$$= 0 \ \text{(as required)}$$

Thus for reversible process net heat had to be absorbed from surroundings in step 5 ($V_5 \rightarrow V_1$) with $\Delta S_{surr} = -\Delta S_{5\rightarrow 1}$ and for system (gas + blocks):

$$\Delta S_{gas+blocks} = C_B \ln\left(\frac{T_F}{T_H}\right) + C_B \ln\left(\frac{T_F}{T_L}\right)$$

$$with\ q_{rev} = T_H \left(C_B \ln\left(\frac{T_F^{\,2}}{T_H T_L}\right)\right) \qquad \text{from the step V isothermal reversible volume change}$$

$$q_{irrev} = 0$$

$$\Delta S_{gas+blocks} = \int \frac{đ\, q_{rev}}{T} > \int \frac{đ\, q_{irrev}}{T}$$