## Some comments on the

# Van der Waals equation of state

## various equations of state (Raff Table 1.2)

### Table 1.2 Some commonly used equations of state

| Equation of State | Functional Form  | Number of Parameters |
|-------------------|--|----------------------|
| Ideal gas         | $PV_m = RT$  | 0                    |
| van der Waals     | $(V_m - b)(P + a/V_m^2) = RT$  | 2                    |
| Dieterici         | $P(V_m - b)\exp[a/RTV_m] = RT$   | 2                    |
| Berthelot         | $(V_m - b)(P + a/TV_m^2) = RT$   | 2                    |
| Virial            | $P = RT \left[ V_m^{-1} + \sum_{n=2}^{\infty} C_n(T) V_m^{-n} \right]$ | œ                    |
| Beattie-Bridgman  | $PV_m^2 = (1 - \gamma)RT(V_m + \beta) - \alpha,$                       | 5                    |
|                   | with $\gamma = c_o/T^3 V_m$  |                      |
|                   | $\beta = b_o[1 - b/V_m]$ , and   |                      |
|                   | $\alpha = a_o[1 + a/V_m]$  |                      |
| Redlich-Kwong     | $P = \frac{RT}{(V_m - b)} - \frac{a}{T^{1/2}V_m(V_m + b)}$             | 2                    |
| Reichsanstalt     | $PV = RT + AP + BP^2 + CP^3 + \cdots$                                  | œ                    |
|                   |  |                      |

$$P_{ideal} \qquad \overline{V}_{ideal} = RT$$

$$\left(P + \frac{a}{\overline{V}^2}\right)\left(\overline{V} - b\right) = RT$$

interpretation of parameters:

b is correction for actual volume of atoms/molecules

- volume available to molecules  $(\overline{V}-b)$  smaller than  $\overline{V}$
- b is associated with repulsive forces

van der Waals equation

$$P_{ideal} \qquad \overline{V}_{ideal} = RT$$

$$\left(P + \frac{a}{\overline{V}^2}\right)\left(\overline{V} - b\right) = RT$$

interpretation of parameters:

a is correction for attractive forces of atoms/molecules

- a is associated with attractive forces
- $P_{bulk} \equiv P_{ideal}$  is greater than  $P \equiv P_{meas}$  measured at surface

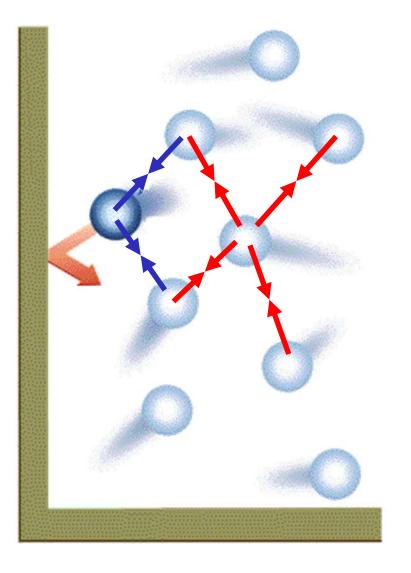
$$\mathsf{P}_{\mathsf{bulk}} = \left( P_{\mathsf{meas}} + \frac{a}{\overline{V}^2} \right) \qquad P_{\mathsf{meas}} \to P_{\mathsf{bulk}} \quad \overline{V} \to \infty$$

5

#### heuristic justification for attractive constant a

- asymmetric attractive forces for molecule at surface
- molecule at surface has less momentum less than molecule in bulk
- P≡P<sub>meas</sub> < P<sub>bulk</sub>

• 
$$\mathsf{P}_{\mathsf{bulk}} = \left(P + \frac{a}{\overline{V}^2}\right)$$



## van der Waals equation

$$\left(P+\frac{a}{\overline{V}^2}\right)\left(\overline{V}-b\right)=RT$$

|      | Table 1.1 | van der Waal                            |                        |                |
|------|-----------|---|------------------------|----------------|
|      | Gas       | <b>b</b> ( <b>L mol</b> <sup>-1</sup> ) | $a (L^2 bar mol^{-2})$ |                |
| ? 🕻  | He        | 0.0238                                  | 0.0346                 | 5              |
|      | Ne        | 0.01672                                 | 0.208                  | polarizability |
| size | Ar        | 0.03201                                 | 1.355                  |                |
|      | Kr        | 0.0396                                  | 2.325                  |                |
| ·    | Acetylene | 0.0522                                  | 4.516                  |                |
|      | $N_2$     | 0.0387                                  | 1.37                   | polarity       |
|      | $H_20$    | 0.03049                                 | 5.537                  |                |
|      | $CO_2$    | 0.04286                                 | 3.658                  |                |
|      |           |   |                        |                |

Source: Handbook of Chemistry and Physics, 78th edition, CRC Press, Boca Raton, Fl, 1997–98 a little trash talk on VDW eqn, but instructive !!!

#### Validity

However, the Van der Waals model is not appropriate for rigorous quantitative calculations, remaining useful only for **teaching** and qualitative purposes.<sup>[1]</sup>

Nowadays, Eq. 2.9 belongs to "**pedagogical physics**:" it is the simplest equation that illustrates several important concepts, but its accuracy is not satisfactory.