

## Chemistry 163B

$q_{\text{rev}}$  , Clausius Inequality and calculating  
 $\Delta S$  for ideal gas P,V,T changes (HW#6)

Challenged Penmanship  
Notes

## *statements of the Second Law of Thermodynamics*

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1. Macroscopic properties of an isolated system eventually assume constant values (e.g. pressure in two bulbs of gas becomes constant; two block of metal reach same T) [*Andrews. p37*]
2. It is impossible to construct a device that operates in cycles and that converts heat into work without producing some other change in the surroundings. *Kelvin's Statement [Raff p 157]; Carnot Cycle*
3. It is impossible to have a natural process which produces no other effect than absorption of heat from a colder body and discharge of heat to a warmer body. *Clausius's Statement, refrigerator*
4. In the neighborhood of any prescribed initial state there are states which cannot be reached by any adiabatic process  
~ *Caratheodory's statement [Andrews p. 58]*

## four steps to exactitude

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$$I. \quad \varepsilon_{\text{CARNOT}[ideal\ gas]} = \frac{-w_{total}}{q_U} = 1 - \frac{T_L}{T_U} = 1 + \frac{q_L}{q_U}$$

$$II. \quad \varepsilon_{\text{ANY REVERSIBLE 'TWO TEMPERATURE' MACHINE}} = \varepsilon_{\text{CARNOT}[ideal\ gas]}$$

or else violation of 2nd Law

$$III. \quad \oint_{\text{cycle}} \frac{dq_{rev}}{T} = 0 \quad \text{eqn 5.11 E \& R; demonstrated for ideal gas Carnot;}$$

general proof for two temperature reversible cycle;  
see "a REALLY BIG RESULT" last lecture

(Dickerson p.155; Raff p.162 - 163)

$$IV. \quad \oint_{\text{cycle}} \frac{dq_{rev}}{T} = 0 \quad \text{for any reversible cyclic process}$$

figure 5.4 E & R

(Dickerson pp.156 - 159, Raff pp.163 - 164)

## *Entropy*

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$dS = \frac{\bar{d}q_{rev}}{T}$  is an exact differential

S is a state function

## *goals of lecture*

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1. Relate  $\Delta S$  and  $q_{\text{irrev}}$
2. Calculate  $\Delta S$  for P,V, T changes of ideal gas (HW#6)
  - a. using REVERSIBLE path ( $q_{\text{rev}}$ ) [***even for irreversible processes***]
  - b. using partial derivatives of S with respect to P, V, T [*a look ahead*]

## entropy and heat for actual (irreversible processes): $q_{irrev}$

an *irreversible* (actual) **cyclic** engine  $\epsilon_{irrev}$  coupled with a Carnot heat pump of  $\epsilon_C$  **will not** violate 2<sup>nd</sup> Law if  $\epsilon_{irrev} < \epsilon_C$  (viz section, HO#25 SL 29;HO #28)

$$\begin{aligned} \Delta U_{cyclic} &= 0 \\ -W_{total} &= q_U + q_L \\ &\text{for both rev} \\ &\text{and irrev} \end{aligned}$$

**BUT** what about  $q_{irrev}$  with  $\epsilon_{irrev} < \epsilon_C$  ??

$$\epsilon_{irrev} = \left( \frac{-w_{total}}{q_U} \right)_{irrev} = \left( \frac{q_U + q_L}{q_U} \right)_{irrev} = 1 + \frac{(q_L)_{irrev}}{(q_U)_{irrev}} < 1 - \frac{T_L}{T_U} = \epsilon_{reversible}$$

$$\frac{(q_L)_{irrev}}{(q_U)_{irrev}} < -\frac{T_L}{T_U}$$

$$\Delta S_{cyclic\ engine} \begin{matrix} \text{(reversible or irreversible)} \\ \end{matrix} = \frac{(q_U)_{rev}}{T_U} + \frac{(q_L)_{rev}}{T_L} = 0$$

$$\frac{(q_L)_{irrev}}{T_L} + \frac{(q_U)_{irrev}}{T_U} < 0 = \Delta S_{cyclic\ engine\ (reversible\ or\ irreversible)}$$

$$\frac{\bar{d}q_{rev}}{T} = dS$$

$$\frac{\bar{d}q_{irrev}}{T} < dS$$

$$\frac{\bar{d}q}{T} \leq dS$$

## 2<sup>nd</sup> Law of Thermodynamics in terms of entropy

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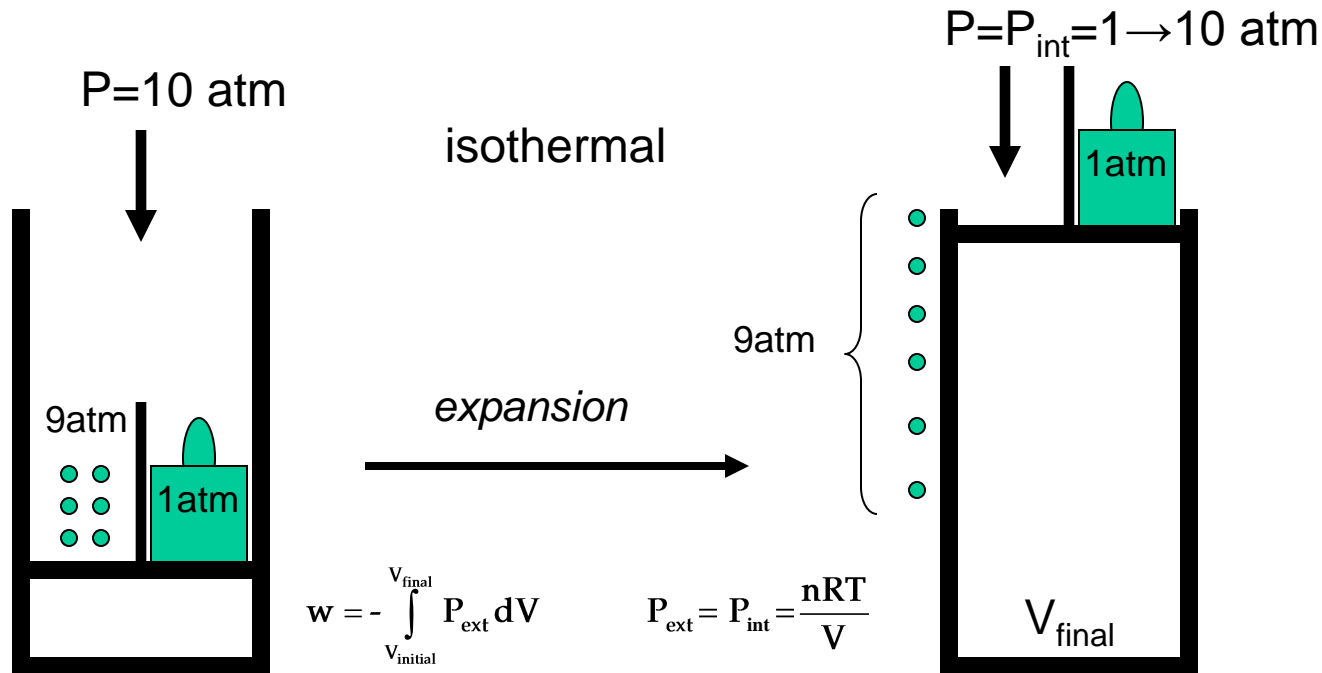
- **S is a STATE FUNCTION**

- $$\Delta S = \int_{rev} \frac{\vec{dq}_{rev}}{T} > \int_{irrev} \frac{\vec{dq}_{irrev}}{T}$$

**E&R eqn 5.33 Clausius inequality**

### Lecture 3:

## Pressure-Volume work reversible isothermal expansion; $P_{ext}=P_{int}$



1 mole  
300K  
10 atm  
 $V_1$

$$w = - \int_{V_{initial}}^{V_{final}} P_{ext} dV$$

$$P_{ext} = P_{int} = \frac{nRT}{V}$$

$$w = - \int_{V_{initial}}^{V_{final}} \frac{nRT}{V} dV = - nRT \int_{V_1}^{V_2} \frac{1}{V} dV = - nRT \ln \frac{V_2}{V_1}$$

$$V_2 = \frac{300K R (1mol)}{1 \text{ atm}} \quad V_1 = \frac{300K R (1mol)}{10 \text{ atm}}$$

$$w = - (300 \text{ K-mol}) R \ln \frac{10 \text{ atm}}{1 \text{ atm}}$$

$$w = -5743 \text{ J} = -5.743 \text{ kJ}$$

(more work done ON surroundings

by reversible than irreversible;  $w_{irrev} = -2.244 \text{ kJ}$ )

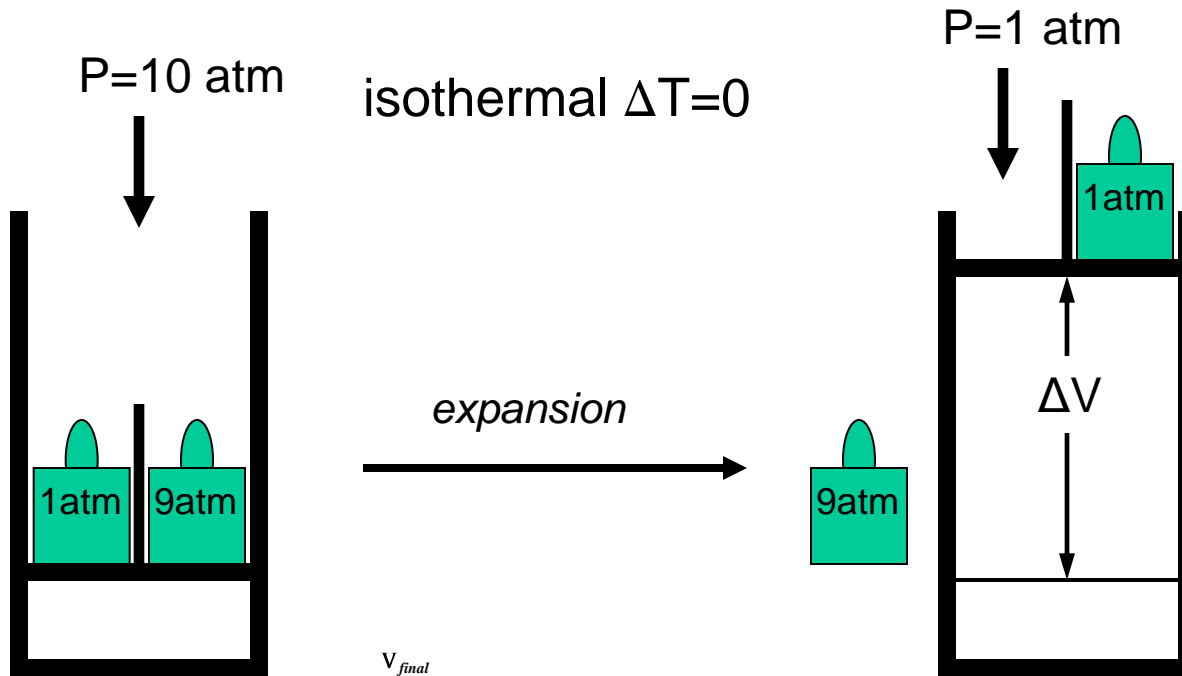
1 mole  
300K  
1 atm  
 $V_2$

$$q_{rev} = -w = +5.743 \text{ kJ}$$



### Lecture 3:

## Isothermal expansion: $P_{ext} = \text{const}$ ideal gas (irreversible)



1 mole  
300K  
10 atm  
 $V_1$

$$w = - \int_{V_{initial}}^{V_{final}} P_{ext} dV$$

$$PV = nRT$$

$$w = -1 \text{ atm} (V_2 - V_1)$$

$$V_2 = \frac{300\text{K} R (1\text{mol})}{1 \text{ atm}} \quad V_1 = \frac{300\text{K} R (1\text{mol})}{10 \text{ atm}}$$

$$w = -(300 \text{ K-mol}) R \left( \frac{1}{1 \text{ atm}} - \frac{1}{10 \text{ atm}} \right)$$

$$w = -2244 \text{ J} = -2.244 \text{ kJ}$$

(- sign implies net work done ON surrounding)

1 mole  
300K  
1 atm  
 $V_2$

$$q_{irrev} = -w = +2.244 \text{ kJ}$$

## EXAMPLE from early lectures: isothermal expansion

$$(P_1=10 \text{ atm}, T_1=300\text{K}, V_1) \rightarrow (P_2=1 \text{ atm}, T_2=300\text{K}, V_2)$$

initial → final

same initial and final

$$\int \frac{\bar{d}q}{T} = \frac{q}{T} \quad \text{for isothermal process} \quad \Delta S$$

$$P_{\text{ext}} = P_{\text{int}}; \quad q_{\text{rev}} = 5743 \text{ J} \quad \frac{q}{T} = 19.14 \text{ J K}^{-1} \quad ?$$

$$P_{\text{ext}} = \text{const } 1 \text{ atm}; \quad q_{\text{irrev}} = 2244 \text{ J} \quad \frac{q}{T} = 7.48 \text{ J K}^{-1} \quad ?$$

$$\Delta S = \int_{\text{initial}}^{\text{final}} \frac{\bar{d}q_{\text{rev}}}{T}$$

*some reversible path*

to calculate  $\Delta S$  must use **reversible** path initial  $\rightarrow$  final

$$\Delta S_{\text{universe}} \geq 0$$

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soon :

$$\Delta S_{\text{system}} + \Delta S_{\text{surroundings}} = \Delta S_{\text{universe}} \geq 0$$

***disorder increases***

## calculating entropy (see summary on review handout)

- Thermal properties of entropy and entropy calculations

- $dS = \frac{dq_{rev}}{T}$ ;  $\Delta S = \int \frac{dq_{rev}}{T}$ ;  $\oint \frac{dq_{rev}}{T} = 0$

- $\Delta S \geq \int \frac{dq}{T}$ ;  $0 \geq \oint \frac{dq}{T}$ ; (= for reversible process; > for spontaneous ['real'] process)

- $\Delta S_{total \equiv universe} = \Delta S_{system} + \Delta S_{surroundings} \geq 0$

S is a state function; dS is an exact differential  
Dependence of S on

- T:  $\left(\frac{\partial \bar{S}}{\partial T}\right)_V = \frac{\bar{C}_V}{T}$ ;  $\left(\frac{\partial \bar{S}}{\partial T}\right)_P = \frac{\bar{C}_P}{T}$

- P:  $\left(\frac{\partial \bar{S}}{\partial P}\right)_T = -\left(\frac{\partial \bar{V}}{\partial T}\right)_P$

- V:  $\left(\frac{\partial \bar{S}}{\partial \bar{V}}\right)_T = \left(\frac{\partial P}{\partial T}\right)_T$

- Phase:  $\Delta S = \frac{\Delta H_{equilibrium\ phase\ change}}{T_{equilibrium\ phase\ change}}$

- Calculation of entropy changes for changes in P, V, T, phase

- Third Law and calculations using Third Law Entropies:  $\bar{S}^o(T)$

- $\Delta S_{reaction}^o(T) = \sum_i \nu_i \bar{S}_i^o(T)$

- Entropy of mixing:  $\Delta S = -n_{total} R \sum_i X_i \ln X_i$  where  $X_i = \frac{n_i}{n_{total}}$

look ahead -  $\Delta S$  for changes in  $T, V$ ; (always  $\Delta S = \int_{initial}^{final} \frac{dq_{rev}}{T}$  )

also:

$S(T, V)$ :

$$dS = \left( \frac{\partial S}{\partial T} \right)_V dT + \left( \frac{\partial S}{\partial V} \right)_T dV$$

coming very soon

$$\left( \frac{\partial S}{\partial T} \right)_V = \frac{n\bar{C}_v}{T} \quad \left( \frac{\partial S}{\partial V} \right)_T = \left( \frac{\partial P}{\partial T} \right)_V$$

so:  $dS = \frac{n\bar{C}_v}{T} dT + \left( \frac{\partial P}{\partial T} \right)_V dV$  **always (no  $w_{other}$ , closed system)**

**ideal gas**

$$dS = \frac{n\bar{C}_v}{T} dT + \frac{nR}{V} dV \quad \Delta S = \int_{rev \text{ const } V \text{ path}} \frac{n\bar{C}_v}{T} dT + \int_{rev \text{ const } T \text{ path}} \frac{nR}{V} dV$$

$$\Delta S = n\bar{C}_v \ln \left( \frac{T_{final}}{T_{initial}} \right) + nR \ln \left( \frac{V_{final}}{V_{initial}} \right) \quad \text{E\&R eqn 5.18}$$

$q_{rev}$   $T$  vary  
const  $V$  path

$q_{rev}$   $V$  vary  
const  $T$  path



*End of Lecture*

## Return of Midterm #1

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can't wait ?  
*pick up exam from TA  
in section or during  
office hours*

**Tianyu  
Secs B and E**

**FRONT OF ROOM**

**Gabe  
Secs A and D**



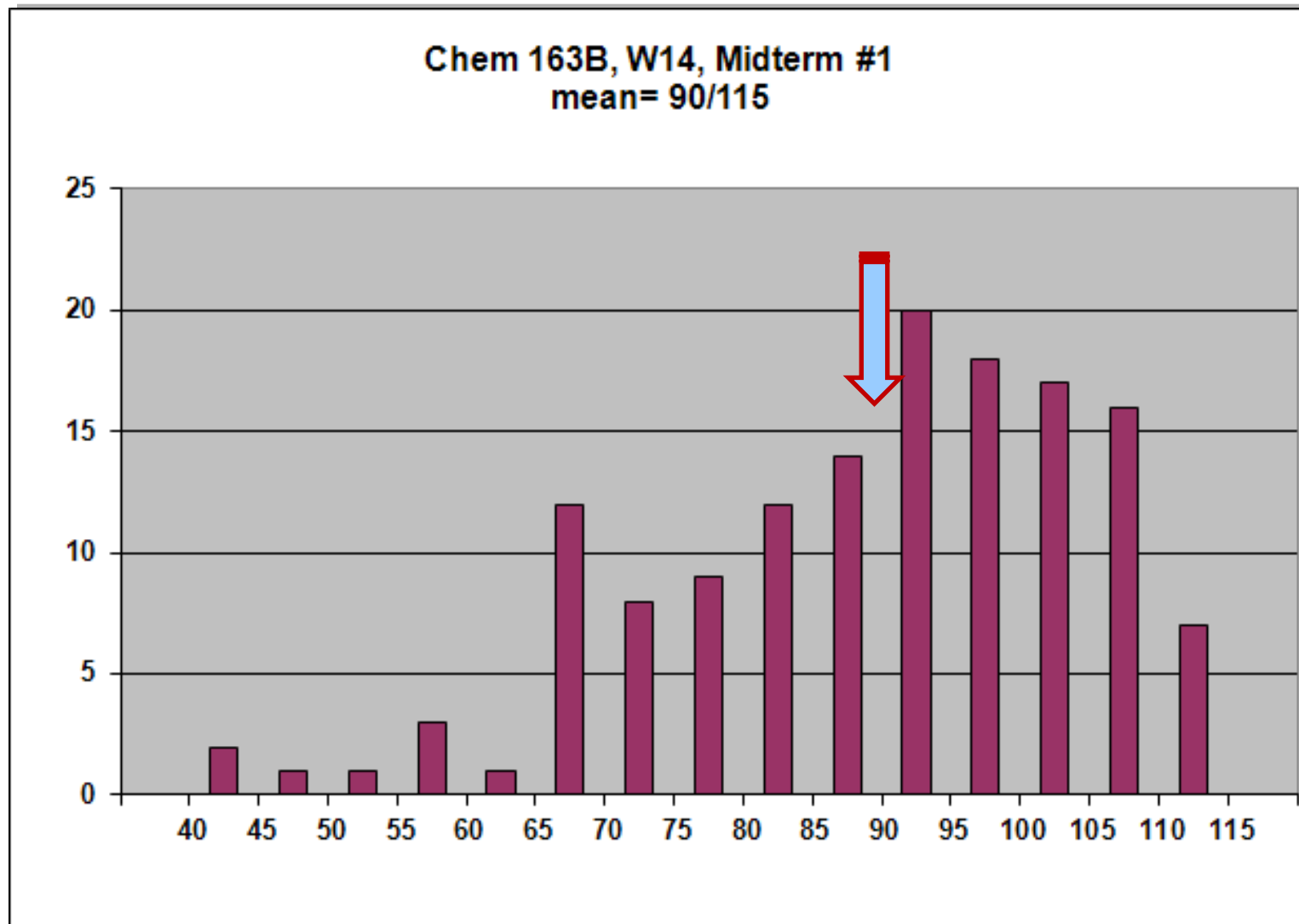


## *midterm#1 return: the scene*

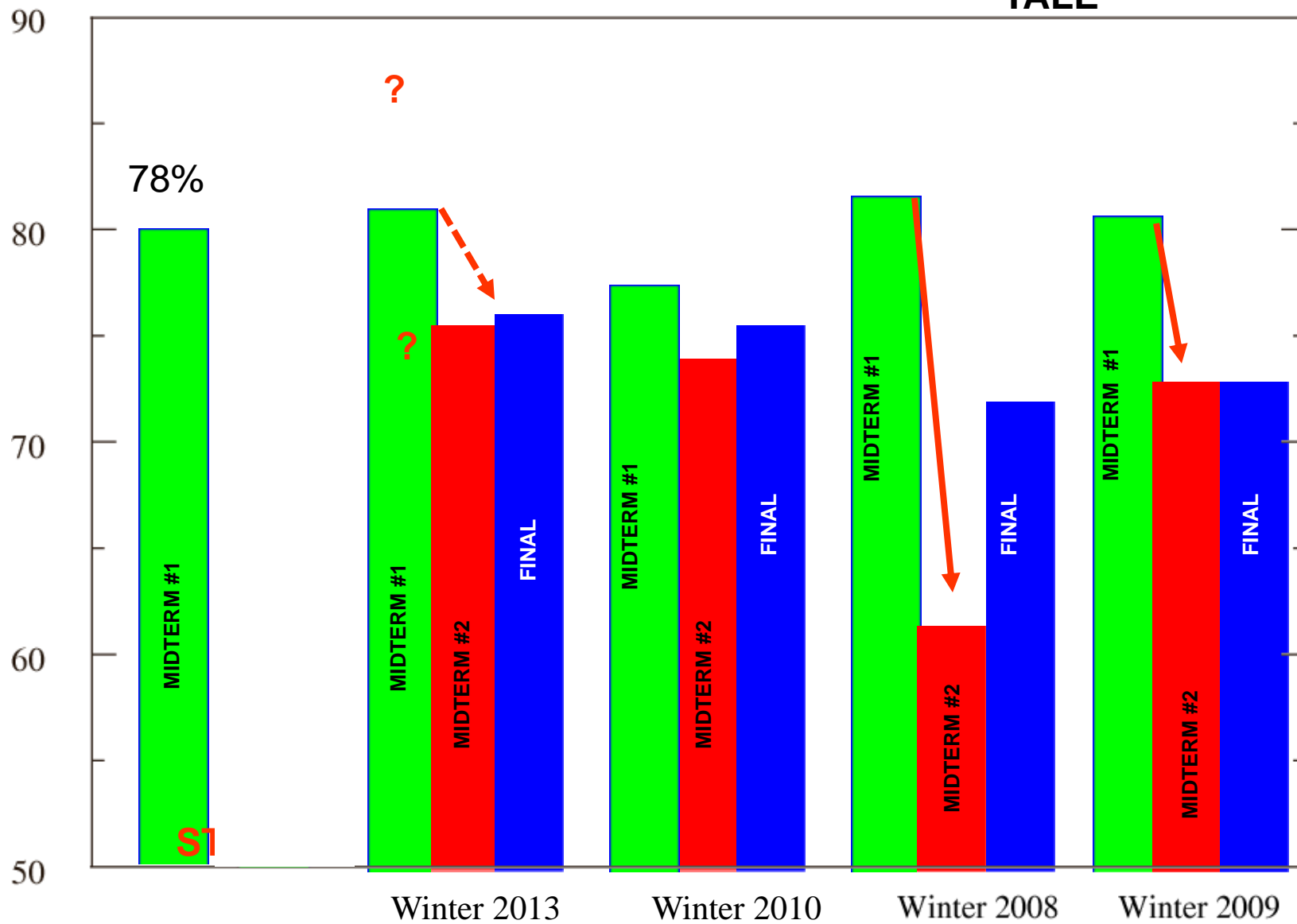
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- no grades on individual midterms (**DON'T ASK !!**)
- class continues to do well, near class average  $\Rightarrow$  B-
- TAs and Amy have keys- speak with them about any questions on exam
- notes on exams from E.S. ( me)
  - ❖ **“Let me know who you are”- props for good scores**
  - ❖ **“Need to do better”- I’m happy to meet with you**
  - ❖ **“ “- can always do better, and we do care about you too !!**

## Midterm #1 Winter 2014

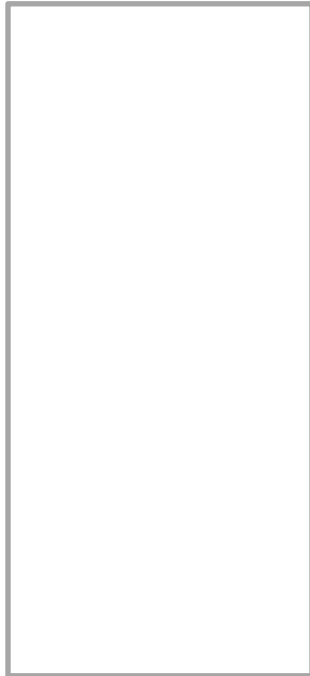


- total pts for qtr (**115** + 138 + 162 + **46**) = **115** + 347 = 464



# Return of Midterm #1

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can't wait ?  
*pick up exam from TA  
in section or during  
office hours*



**Tianyu  
Secs B and E**

**FRONT OF ROOM**

**Gabe  
Secs A and D**

