Chemistry 163B

μ_i and $\Delta\mu_{reaction}$

Activity

Equilibrium

goals

- Derive equilibrium and spontaneity criteria applying multicomponent thermodynamic relationships; i.e chemical potential ($\Delta\mu_{reaction}$)
- Define concentration dependence of μ in terms of **activity** (fugacity) of 'real' gases, actual solutes
- Apply **activity** to equilibrium K_{eq}
- Derive how to obtain fugacity if **REAL** gas

equilibrium in terms of $\Delta\mu$

$$n_A\, A + n_B\, B \longrightarrow n_C\, C + n_D\, D$$

$\nu_A = -n_A$ $\nu_B = -n_B$ $\nu_C = +n_C$ $\nu_D = +n_D$

$d\xi$ is extent of reaction
$d\xi > 0$ forward reaction
$d\xi < 0$ reverse reaction

$dn_i = \nu_i\, d\xi$
$dn_i > 0$ substance *i* increases
$dn_i < 0$ substance *i* decreases

equilibrium in terms of $\Delta\mu$

$$dG_{T,P} < 0 \text{ spontaneous}$$

$$dG_{T,P} = 0 \text{ equilibium}$$

whole pot of mixed reactants and products

$$dG = -SdT + VdP + \sum_{i=1}^{N} \mu_i dn_i$$

$$dn_i = \nu_i d\xi$$

$$dG = -SdT + VdP + \sum_{i=1}^{N} \mu_i \nu_i \, d\xi$$

$$dG_{T,P} = \left(\sum_{i=1}^{N} \mu_i \nu_i \right) d\xi \leq 0$$

equilibrium in terms of Δμ

$$dG_{T,P} = \underbrace{\left(\sum_{i=1}^{N} \mu_i \nu_i\right)}_{?} d\xi \le 0$$

$$dG_{T,P} = \underbrace{\left(\sum_{i=1}^{N} \mu_i \nu_i\right)}_{\Delta\mu_{reaction}} d\xi \le 0$$

$$dG_{T,P} = \Delta\mu_{reaction} d\xi \le 0$$

$\Delta\mu_{reaction} < 0$ forward reaction spontaneous ($d\xi > 0$)

$\Delta\mu_{reaction} > 0$ reverse reaction spontaneous ($d\xi < 0$)

$\Delta\mu_{reaction} = 0$ equilibrium

just like ΔG !!!

NOTE: μ_I IS INTENSIVE ($J\ mol^{-1}$)
$\Delta\mu_{reaction}$ IS EXTENSIVE (J)

concentration dependence of μ_i

ideal gas, one component (pure substance)

$$\bar{G} = \bar{G}^\circ + RT\ ln\left(\frac{P}{1\,bar}\right)$$

led to

$$\Delta G_{reaction} = \Delta G^\circ_{reaction} + \underline{R}T\ ln(Q_P)$$

what about if other species present?

$$\mu_i = \mu_i^\circ + RT\ ln\left(\frac{P_i}{1\ bar}\right)$$

HANDOUT #48

$$\Delta\mu_{reaction} = \Delta\mu^\circ_{reaction} + \underline{R}T\ ln Q_P$$

$$\Delta\mu^\circ_{reaction} = \sum_i \nu_i \mu_i^\circ \qquad Q_P = \prod_i \left(\frac{P_i}{1\,bar}\right)^{\nu_i}$$

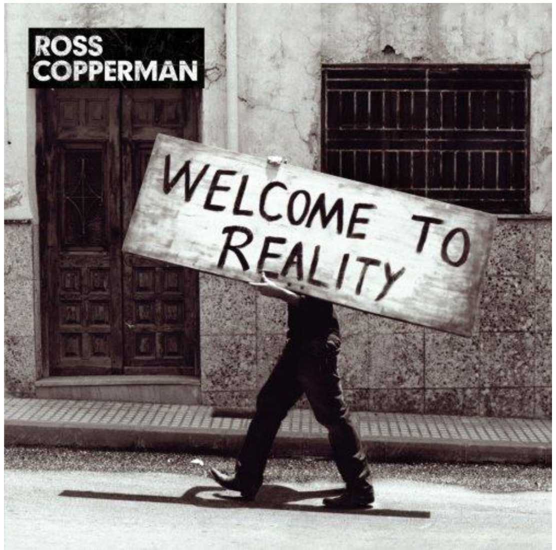
yada -yada- yada: and so forth for $\Delta\mu_{reaction}$

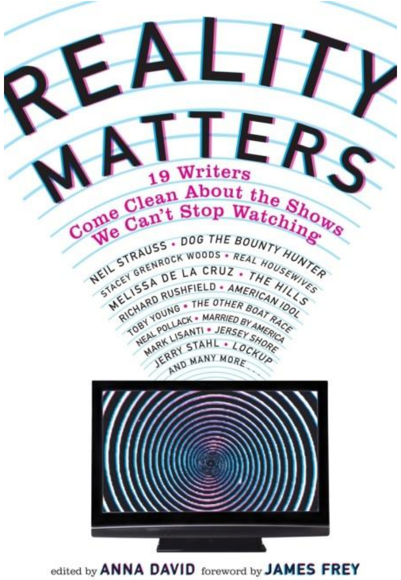
$$\Delta\mu^{\circ}_{reaction} = -\underline{R}T \ln K_P$$

$$\left(\frac{\partial \frac{\Delta\mu}{T}}{\partial T}\right)_P = -\frac{\Delta H}{T^2} \quad \text{where} \quad \Delta H = \sum_i \nu_i \bar{H}_i = \sum_i \nu_i \left(\frac{\partial H}{\partial n_i}\right)_{T,P,n_j \neq n_i}$$

$$\left(\frac{\partial \ln K}{\partial T}\right)_P = \frac{\Delta H^{\circ}}{\underline{R}T^2} \quad \text{where} \quad \Delta H^{\circ} = \sum_i \nu_i \bar{H}^{\circ}_i$$

correcting for REALITY (activity and fugacity)





correcting for REALITY (activity and fugacity)

- will define activity and fugacity coefficients **γ_i's** that provides corrections for the deviation of chemical potential from the ideal gas and solute concentration dependence

- activity and fugacity coefficients are obtained from experimental measurements on REAL systems or by theory (Debye-Huckel)

more general μ_i and corrections for non-ideality (~E&R eqn 9.50, p227)$_{3rd}$

$$\mu_i(T) = \mu_i^\circ(T) + RT\ ln\left(a_i / a_i^0 \right)$$

standard conditions

how μ_i 'really' changes in going from standard conditions to actual conditions

$$a_i = activity\ of\ component\ i$$

$$a_i = \gamma_i \times [ideal\ measure\ of\ pressure, concentration, etc]$$

γ_i is activity coefficient, a correction for non-ideality

$$a_i^o = 1\quad unit\quad (bar, molar, etc)$$

more general μ_I and corrections for non-ideality

$$\Delta\mu_{reaction} = \Delta\mu^{o} + \underline{R}T\ \ln Q$$

where now Q is written in terms of activities

$$Q = \prod_i \left(\frac{a_i}{a_i^{\circ}}\right)^{\bar{\nu}_i} \qquad Q = \underbrace{\frac{(a_C)^{\bar{n}_C}(a_D)^{\bar{n}_D}}{(a_A)^{\bar{n}_A}(a_B)^{\bar{n}_B}}}$$

unitless

dropped the $a_i^{\circ} = 1$ ***'unit'***
but Q is 'unitless'

1. GASES:

$$a_i = f_i = \gamma_i \; P_i$$

partial pressure

fugacity of gas

fugacity coefficient

$$Q = \frac{(a_C)^{\bar{n}_C}(a_D)^{\bar{n}_D}}{(a_A)^{\bar{n}_A}(a_B)^{\bar{n}_B}}$$

$$Q = \frac{\left(\gamma_C \frac{P_c}{1bar}\right)^{\bar{n}_C}\left(\gamma_D \frac{P_D}{1bar}\right)^{\bar{n}_D}}{\left(\gamma_A \frac{P_A}{1bar}\right)^{\bar{n}_A}\left(\gamma_B \frac{P_B}{1bar}\right)^{\bar{n}_B}}$$

$$Q = \left(\frac{\gamma_C^{\bar{n}_C}\gamma_D^{\bar{n}_D}}{\gamma_A^{\bar{n}_A}\gamma_B^{\bar{n}_B}}\right)\frac{\left(\frac{P_C}{1bar}\right)^{\bar{n}_C}\left(\frac{P_D}{1bar}\right)^{\bar{n}_D}}{\left(\frac{P_A}{1bar}\right)^{\bar{n}_A}\left(\frac{P_B}{1bar}\right)^{\bar{n}_B}} = \underline{\underline{\gamma}}\, Q_P$$

correction for non-ideality

ideal gas Q_P

other conventions for activities

2. pure solids and liquids

$$\mu_i(T,P) \approx \mu_i^\circ(T, P = 1bar)$$

$$\left(\frac{\partial \mu_i}{\partial P}\right)_T = \bar{V}_i \quad \textit{(small for liquid or solid)}$$

so $a_i \approx 1$ *for pure solid or liquid*

[unless extreme pressure]

EXPERT

other conventions for activities

3. solutes in solutions

$$a_i = \gamma_i [I]$$ ← concentration of I, usually molar but may be X_i

WHY CONCENTRATION FOR SOLUTES IN SOLUTION ?
HAVE PROVEN ,P_i=$X_i P_{TOTAL}$ ⇨ Q FOR IDEAL GAS.
SOON WILL JUSTIFY [I] =X_i ⇨ Q FOR ,SOLUTES IN IDEAL SOLNS.

activity coefficient γ_i corrects 'ideal' measure of 'concentration

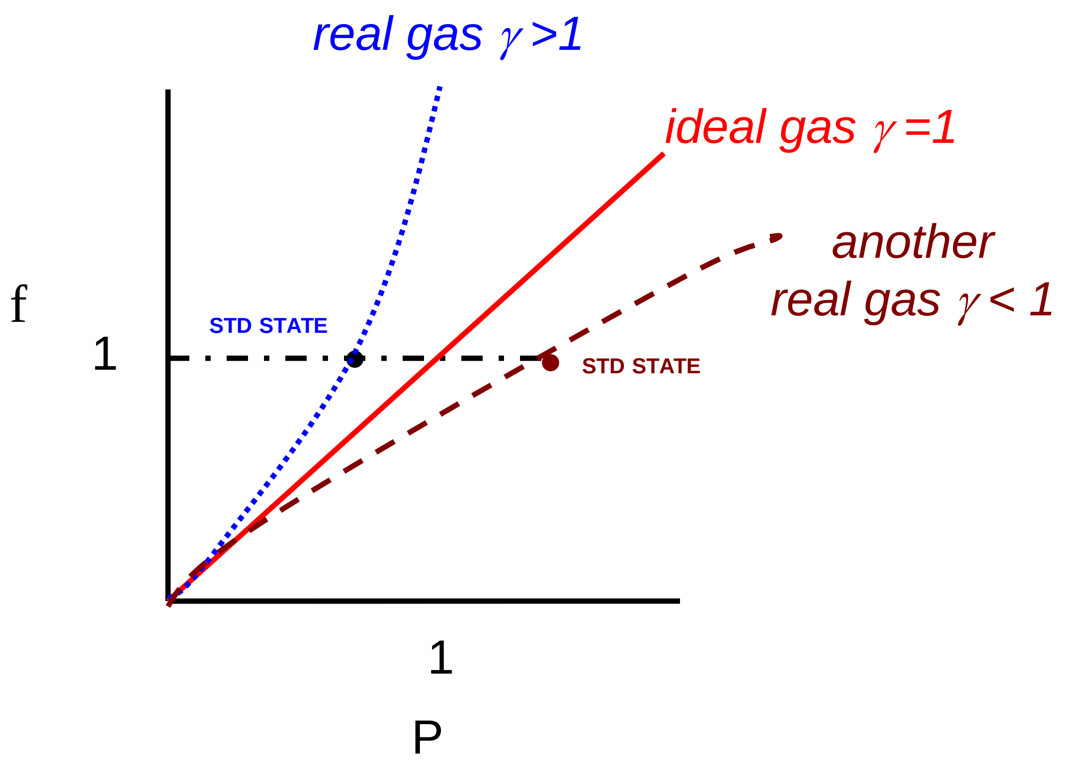
if "activity coefficients unity"

$$a_i = [I] \qquad a_i \equiv f_i = P_i \qquad a_i = 1$$

solute *gas* *pure liquid or solid*

HW#8 γ=1 except prob. 41* and 43.

*how to evaluate activity (fugacity) coefficients for real gases (HW8 #*41)*



how to evaluate activity (fugacity) coefficients for real gases

$$\left(\frac{\partial \mu}{\partial P}\right)_{T,n} = \bar{V} \quad and$$

$$\mu = \mu^{\circ} + RT\ ln\ f$$

need in a moment

$$\left(\frac{\partial \mu}{\partial P}\right)_{T,n} = RT\left(\frac{\partial \ln f}{\partial P}\right)_{T,n}$$

$$so \quad RT\left(\frac{\partial \ln f}{\partial P}\right)_{T,n} = \bar{V}$$

expression for $d\ln\left(\frac{f}{P}\right)$ will prove useful

$$\left(\frac{\partial ln\left(\frac{f}{P}\right)}{\partial P}\right)_{T,n} = \left(\frac{\partial\left(ln\ f\ \text{-}\ ln\ P\right)}{\partial P}\right)_{T,n} = \left(\frac{\partial \ln f}{\partial P}\right)_{T,n} - \left(\frac{\partial \ln P}{\partial P}\right)_{T,n}$$

$$= \frac{1}{RT}\bar{V} - \frac{1}{P} = \frac{1}{RT}\left(\bar{V} - \frac{RT}{P}\right)$$

how to evaluate activity (fugacity) coefficients for real gases

$$\int_{P_1}^{P} d\left(ln \frac{f}{P'} \right) = \int_{P_1}^{P} \frac{1}{RT} \left(\bar{V} - \frac{RT}{P'} \right) dP'$$

$$ln\left(\frac{f(P)}{P} \right) - ln\left(\frac{f(P_1)}{P_1} \right) = \frac{1}{RT} \int_{P_1}^{P} \left(\bar{V} - \frac{RT}{P'} \right) dP'$$

$$ln\left(\frac{f(P)}{P} \right) = ln\left(\frac{f(P_1)}{P_1} \right) + \frac{1}{RT} \int_{P_1}^{P} \left(\bar{V} - \frac{RT}{P'} \right) dP'$$

$P_1 \longrightarrow 0$

$$\frac{f(P_1)}{P_1} \rightarrow 1$$

$$ln\left(\frac{f(P_1)}{P_1} \right) \rightarrow 0$$

$$\left(\bar{V}_{ACTUAL} - \bar{V}_{IDEAL\ GAS} \right)$$

how to evaluate activity (fugacity) coefficients for real gases (eqn 7.20 E&R and HW8 #*41)

$$\ln f(P) = \ln P + \frac{1}{RT}\int_{P_1\to 0}^{P}\left(\bar{V} - \frac{RT}{P'}\right)dP' = \ln P + \frac{1}{RT}\int_{P_1\to 0}^{P}\left(\bar{V}_{ACTUAL} - \bar{V}_{IDEAL\,GAS}\right)dP'$$

$$\ln\left(\frac{f(P)}{P}\right) = \ln(\gamma) = \frac{1}{RT}\int_{P_1\to 0}^{P}\left(\bar{V} - \frac{RT}{P'}\right)dP' = \frac{1}{RT}\int_{P_1\to 0}^{P}\left(\bar{V}_{ACTUAL} - \bar{V}_{IDEAL}\right)dP'$$

$$z = \frac{\bar{V}_{actual}}{\bar{V}_{ideal}} = \frac{P\bar{V}_{actual}}{RT} \quad \text{(compression factor E\&R eqn. 7.6)}$$

$$\ln\gamma = \frac{1}{RT}\int_{P_1\to 0}^{P}\bar{V}_{ideal}(z-1)dP' = \int_{P_1\to 0}^{P}\frac{(z-1)}{P'}dP'$$

HW8 41*

$$\gamma(P,T) = \exp\left[\int_{P_1\to 0}^{P}\frac{z-1}{P'}dP'\right] \quad (E\ \&\ R\ eqn\ 7.21)$$

End of Lecture

activity for solids and liquids ($P_{total} \neq 1$ bar)

$$\mu_i(T) = \mu_i^{\circ}(T) + RT\ ln\left(a_i / a_i^0 \right)$$

$$\left(a_i / a_i^0 \right) = e^{\frac{\mu_i(T) - \mu_i^{\circ}(T)}{RT}}$$

$$\left(\frac{\partial \mu_i}{\partial P} \right)_{T,n} = \bar{V}_i$$

$$\mu_i(T, P_{total}) = \mu_i^o(T, 1\text{ bar}) + \int_{1bar}^{P_{total}} \bar{V}_i dP \approx \mu_i^o(T, 1\text{ bar}) + \bar{V}_i \left(P_{total} - 1\text{ bar} \right)$$

HW#7 37 (E&R 6.5)

$$\left(a_i / a_i^0 \right)_{l\text{ or s}} \approx e^{\frac{\bar{V}_i (P_{total} - 1\text{ bar})}{RT}} \approx 1 \text{ for } P_{total} \text{ near 1 bar (since } \bar{V} \text{ is small for liquids or solids)}$$

for $\Delta\mu$ at high P_{total} would use this in Q for liquids and solids

Midterm #2, Wednesday 26th February

- Exam WILL COVER material on lectures #8-#16 *(#1-#16)*
  (~through Friday , 21st February) and HW#5-HW#7

- Weeks 4-7 "reviews" on WWW handouts (19th February)

- HW solutions on eCommons (HW#7 Mon, 24th February)

- SAMPLE ('~~practice~~') midterm on eCommons reserve
  Wednesday, 19th February
  (see sample exam for 'relationships provided')

- TAs have KEY for use in sections, tutorials, and office hours
  (on eCommons ~23rd February)

- REVIEW SESSION
  Monday, 24th February, Classroom Unit 1, 7:00-8:00 PM

- Midterm #2 Exam, Wednesday 26th February
  done "in place"
  no ear dingies (iPODs, MP3, etc)
  bring charged calculators
  get here ON TIME