Review Topics the Entire Quarter !!!

In reviewing material one should not memorize the entries as 'plug-in' formulas. But should understand i) how they arise from the basic laws of thermodynamics; ii) what are the limits on their generality and under what special conditions they apply; and iii) how they qualitatively reflect the 'logic' of the natural world and, in some cases, the underlying properties of molecules and matter.

- Mathematical techniques (to be employed throughout course as needed)
 - Exact and inexact differentials
 - State functions
- Equations of state
 - Thermodynamic variables (P, V, T, variables for w_{other}) and ideal gas thermometer to operationally measure T
 - P=F/A
 - System and surroundings
 - Ideal Gas
 - PV=nRT
 - E= (3/2) nRT (monatomic; no "internal" structure, translational kinetic energy in 3 dimensions)
 - Approximation for real gasses
 - Van der Waals $\left(P \frac{a}{\overline{V}^2}\right) \left(\overline{V} nb\right) = RT$; physical meaning of a and b
 - Other approximate equations of state exist (e.g. Virial power series expansion)
- Work
 - w= force displacement
 - \circ \vec{a} w inexact differential, depends on path
 - $\circ \quad \vec{d} w = -P_{ext} dV \quad (P-V work)$
 - \circ w = $\int dw$
 - w= $-P_{ext} \Delta V$ (Pext = constant, wother = 0)
 - $_{\circ}$ w= -n R T In (V_f / V_i) (isothermal, reversible, ideal gas, w_{other} = 0)
- Heat
 - \circ dq inexact differential depends on path
 - $\circ \quad \mathbf{\vec{d}} \mathbf{q} = \mathbf{C} \, dT$
 - $\circ \quad \mathbf{q} = \int \mathbf{C} \, dT$
- First Law of Thermodynamics
 - U = internal energy
 - dU = dq + dw (definition, for closed system)
 - Equivalent statements of First Law of Thermodynamics
 - U is a state function
 - *d*U is an exact differential
 - $\Delta U_{sys} = -\Delta U_{surr}$

- $H \equiv U + (PV)_{internal}$
- H is a state function
- $\circ \quad \Delta H = \Delta U + \Delta (PV) = \Delta U + P_f V_f P_i V_i$
- Differential relationships for state functions

 (ni= number moles of substance i; ov1 ..ovM are "other variables" that might define the energy of the system and be involved in w_{other}. For example charge, position with respect gravitational attraction, etc)
- $\mathbf{d}H(\mathbf{T},\mathbf{P},\mathbf{n}_{1},...,\mathbf{n}_{K},\mathbf{ov}_{1},...,\mathbf{ov}_{M}) = \left(\frac{\partial \mathbf{H}}{\partial \mathbf{T}}\right)_{\mathbf{P},\mathbf{n},\mathbf{ov}} \mathbf{d}\mathbf{T} + \left(\frac{\partial \mathbf{H}}{\partial \mathbf{P}}\right)_{\mathbf{T},\mathbf{n},\mathbf{ov}} \mathbf{d}\mathbf{P} + \sum_{i}^{K} \left(\frac{\partial \mathbf{H}}{\partial \mathbf{n}_{i}}\right)_{\mathbf{T},\mathbf{P},\mathbf{n}_{j\neq i},\mathbf{ov}} \mathbf{d}\mathbf{n}_{i} + \sum_{i}^{M} \left(\frac{\partial \mathbf{H}}{\partial \mathbf{ov}_{i}}\right)_{\mathbf{T},\mathbf{P},\mathbf{n},\mathbf{ov}_{j\neq i}} \mathbf{d}\mathbf{ov}_{i} \mathbf{d}\mathbf{n}_{i} + \sum_{i}^{M} \left(\frac{\partial \mathbf{H}}{\partial \mathbf{ov}_{i}}\right)_{\mathbf{T},\mathbf{P},\mathbf{n},\mathbf{ov}_{j\neq i}} \mathbf{d}\mathbf{v}_{i} \mathbf{d}\mathbf{v$

more commonly (for closed system, $dn_i = 0$; and no work other, $\vec{d} w_{other} = 0$)

- $dU(T, V) = \left(\frac{\partial U}{\partial T}\right)_{V} dT + \left(\frac{\partial U}{\partial V}\right)_{T} dV$
- $dH(T,P) = \left(\frac{\partial H}{\partial T}\right)_{P} dT + \left(\frac{\partial H}{\partial P}\right)_{T} dP$
- Definitions relationships (closed system)
 - $\circ \quad d\mathsf{U} = \vec{a}\,\mathsf{q} \mathsf{P}_{\mathsf{ext}}\,\mathsf{dV} + \vec{a}\,\mathsf{W}_{other}$
 - $\Delta U_V = q_V + \vec{a} w_{other}$ (const V) "heat is ΔU at constant V"

$$\circ \quad n\overline{C}_{V} \equiv \left(\frac{dq}{dT}\right)_{V} \quad \Rightarrow \quad dU_{V} = n\overline{C}_{V} dT + dw_{other} \text{ (const V)}$$

• $dH = dq + V dP + dw_{other}$ (Pext = Pinternal; thus usually for a reversible process; but OK for any infinitesimal process)

• $\Delta H_P = q_P + w_{other}$ (const Pext; Pintial = Pfinal = Pext) "heat is ΔH at constant P"

$$\circ \quad n\overline{C}_{P} \equiv \left(\frac{dq}{dT}\right)_{P} \implies dH_{P} = n\overline{C}_{P} dT + \vec{d} w_{other} \quad (const P)$$

• Derived consequences (closed system, $\vec{d} w_{other} = 0$)

$$dU = n\overline{C}_{V}dT + \left(\frac{\partial U}{\partial V}\right)_{T}dV$$

with $\left(\frac{\partial U}{\partial V}\right)_{T} = \left(\frac{dq}{dV}\right)_{T} - P$

 \circ $dU_V = n \overline{C}_V dT$ (const V, any system)

0

but with $\left(\frac{\partial U}{\partial V}\right)_{T} = 0$ for ideal gas

$$\circ dU = n \overline{C}_v dT$$
 (ideal gas, any path)

$${}^{\circ} \qquad dH = n\overline{C}_{P}dT + \left(\frac{\partial H}{\partial P}\right)_{T}dP$$
$${}^{\circ} \qquad \text{with}\left(\frac{\partial H}{\partial P}\right)_{T} = \left(\frac{dq}{dP}\right)_{T} + V$$

$$_{\circ}$$
 dH_P = n \overline{C}_{P} dT (const P, any system)

since for ideal gas dH = dU + d (PV) = dU + d(nRT) = dU + nR dTand $dU = n \overline{C}_{V} dT$

• Evaluating q, w, ΔU , ΔH for various processes (closed system, no w_{other})

Constant volume

- w = 0
- $q_v = \Delta U_v = n \overline{C}_v \Delta T$
- $\Delta H_v = \Delta U_v + V \Delta P = \Delta U_v + V (P_f P_i)$

Constant pressure

- $q_p = \Delta H_P = n \overline{C}_p \Delta T$
- w = P∆V
- ▲U_P = q + w

- $\bullet \quad \frac{\textit{Ideal gasses}}{\overline{C}_{p}} = \overline{C}_{v} + R$
- $\overline{C}_{V} = \frac{3}{2} \mathbf{R}$ for monatomic ideal gas
- $\Delta U = n \overline{C}_{v} \Delta T$ (any path)
- $\Delta H = n \overline{C}_{P} \Delta T$ (any path) Isothermal path
 - q = w
 - $\Delta U = 0$
 - $\Delta H = 0$
 - w = n R T $\ln \frac{V_f}{V_i}$ = \Box n R T $\ln \frac{P_i}{P_f}$ (reversible path , note sign)
 - $w = -P_{ext}\Delta V$ (constant P_{ext} expansion/compression)

Adiabatic path

- q = 0
- $\Delta U = w = n \overline{C}_v \Delta T$
- $\Delta H = \Delta U + \Delta (PV) = \Delta U + P_f V_f P_i V_i = n \overline{C}_p \Delta T$

•
$$T_1^{\frac{C_v}{R}}V_1 = T_2^{\frac{C_v}{R}}V_2$$
 (adiabatic reversible path)

•
$$P_1 V_1^{\overline{C}_p} = P_2 V_2^{\overline{C}_p}$$
 (adiabatic reversible path, PV γ = constant)

•
$$\frac{T_1^{\frac{\overline{C}_P}{R}}}{P_1} = \frac{T_2^{\frac{\overline{C}_P}{R}}}{P_2}$$
 (adiabatic reversible path)

Thermochemistry •

- $\Delta H_{\text{reaction}} = (H_{\text{products}} H_{\text{reactants}}) (vs \ \Delta H \text{ for physical change})$
- $\circ \Delta H_P = q_P$
- standard states
- definition of $\overline{H}_{f}^{o} \equiv \Delta \overline{H}_{f}^{o}$

•
$$\Delta H^{o}_{reaction} = \sum_{i} v_{i} \overline{H}^{o}_{f}$$
 where v_{i} are stoichiometric coefficients

Hess's Law 0

$$\circ \quad \Delta H_{\text{reaction}} = \Delta U_{\text{reaction}} + \Delta n_{\text{gas}} RT$$

$$\circ \quad \Delta H(T_2) = \Delta H(T_1) + \int_{T_1}^{T_2} \Delta C_P dT$$

- \circ ΔH from bond enthalpies
- Heats of reactions in ionic solution

- Statements of the Second Law of Thermodynamics
 - Macroscopic properties of an <u>isolated system</u> eventually assume constant values (e.g. pressure in two bulbs of gas_becomes constant; two block of metal reach same T) [*Andrews.* p37]
 - It is impossible to construct a device that operates in cycles and that converts heat into work without producing some other change in the surroundings. *Kelvin's Statement [Raff p 157]; Carnot Cycle*
 - It is impossible to have a natural process which produces no other effect than absorption of heat from a colder body and discharge of heat to a warmer body. *Clausius's Statement, refrigerator*
 - In the neighborhood of any prescribed initial state there are states which cannot be reached by any adiabatic process
 - ~ Caratheodory's statement [Andrews p. 58]
 - Important statements regarding entropy and statistical disorder
 - Greater number of microstates, greater disorder
 - Each allowed microstate is equally probable
 - the overwhelming number of microstates correspond to macrostates with almost identical macroscopic properties
 - W, the number of microstates corresponding to the macrostate, is a measure of the DISORDER of the system in that macrostate
 - A system "meanders" through all available microstates; but you are only likely to observe it in one of the overwhelming number that correspond to the equilibrium macrostate
 - W, the number of microstates corresponding to the macrostate, depends on the populations of various energy levels
 - Adiabatic, reversible, work changes the energy of system but not the populations of the energy levels (the disorder, W)
 - Reversible heat transfers change the populations of the energy levels (and thus W)
- Carnot engines (reversible, $q_H > 0$ at T_H ; $q_L < 0$ at T_L ; $w_t < 0$)
 - Demonstration of machine consistent with 2nd Law (i.e. in a cyclic process, heat can be converted to work if and only if heat is returned to surroundings at a lower temperature.

$$\varepsilon = \frac{-W_t}{q_H} = 1 - \frac{T_L}{T_H}$$

• Reverse process corresponds to refrigerators and heat pumps

$$\circ \qquad \sum \frac{q}{T} = \frac{q_H}{T_H} + \frac{q_L}{T_L} = 0$$

- $_\circ~~\epsilon$ of any Carnot engine ('any working substance") has the same dependence on TH and TL
- Any reversible cyclic process can be describes as a sum of (infinitesimal) Carnot cycles, thus the properties of any cyclic process are combinations of those of Carnot cycles
- Thermal properties of entropy and entropy calculations

$$dS = \frac{\vec{d} q_{rev}}{T}; \quad \Delta S = \int \frac{\vec{d} q_{rev}}{T}; \quad \oint \frac{\vec{d} q_{rev}}{T} = 0$$

$$\Delta S \ge \int \frac{\vec{d} q}{T}; \quad 0 \ge \oint \frac{\vec{d} q}{T}; \quad (= for \ reversible \ process; > for \ spontaneous \ ['real'] \ process)$$

$$\circ \qquad \Delta S_{\textit{total=universe}} = \Delta S_{\textit{system}} + \Delta S_{\textit{surroundings}} \geq 0$$

- S is a state function; dS is an exact differential
- Dependence of S on

•
$$T: \left(\frac{\partial \overline{S}}{\partial T}\right)_{V} = \frac{\overline{C}_{V}}{T}; \quad \left(\frac{\partial \overline{S}}{\partial T}\right)_{P} = \frac{\overline{C}_{P}}{T}$$

• $P: \left(\frac{\partial \overline{S}}{\partial P}\right)_{T} = -\left(\frac{\partial \overline{V}}{\partial T}\right)_{P}$
• $V: \left(\frac{\partial \overline{S}}{\partial \overline{V}}\right)_{T} = \left(\frac{\partial P}{\partial T}\right)_{\overline{V}}$
• Phase: $\Delta S = \frac{\Delta H_{equilibrium phase change}}{2}$

$$T_{equilibrium\ phase\ change}$$

- ^o Calculation of entropy changes for changes in P, V, T, phase
- Third Law and calculations using Third Law Entropies: $\overline{S}^{o}(T)$

$$\circ \quad \Delta S^{0}_{reaction}(T) = \sum_{i} \nu_{i} \overline{S}^{0}_{i}(T)$$

- Entropy of mixing: $\Delta S = -n_{total} R \sum_{i} X_{i} \ln X_{i}$ where $X_{i} = \frac{n_{i}}{n_{total}}$
- Qualitative assessment of factors affecting $\overline{S}(T, P, V)$ and $\Delta S_{reaction}$
- Derivation of various thermodynamic relationships
 - mathematical tools
 - properties of exact differentials
 - "dividing through" by 'd?"
 - Euler-Maxwell relationships
 - state functions
 - U=q+w
 - H≡U+PV
 - A≡U-TS
 - G≡H-TS
 - o and their total differentials (no work other, closed systems)

- TdS = $d\bar{q}$ TdS = $d\bar{q}_{rev}$ (P_{ext})_{rev}=P_{int}=P
- dU = TdS PdV U(S,V) internal energy
- dH = TdS + VdP H(S,P) enthalpy
- dA = -SdT PdV A(T,V) Helmholtz free energy
- dG = -SdT + VdP G(T,P) Gibbs free energy
- Free energy defined
 - ∘ A≡U TS
 - o G≡H TS
 - \circ dA= SdT PdV
 - ∘ dG= SdT +VdP
- Free energy: equilibrium (=) and spontaneity (<)
 - $\circ \quad \Delta A_{T,V} \leq 0$
 - $\circ \quad \Delta G_{T,P} \leq 0$
- Relationship of $\Delta G_{T,P} \leq 0$ to 2^{nd} Law
 - $\begin{array}{l} \circ \quad -\frac{\Delta G_{T,P}}{T} = -\frac{\Delta H}{T} + \Delta S \ge 0 \\ \Delta S_{surr} \ge -\frac{\left(\Delta H_{P}\right)_{sys}}{T} \quad \text{disorders surroundings} \\ \circ \quad \Delta S_{sys} \qquad \qquad \text{disorders system} \\ \Delta S_{surr} \quad + \quad \Delta S_{sys} \ge 0 \end{array}$
- Calculation of ΔG for physical processes (expansion of gasses, phase changes, etc.)
- Calculation of $\Delta G_{reaction}$ from ΔG_f or ΔH_f and ΔS or reactants and products
- $\Delta G_{T,P}$ at arbitrary partial pressures (concentrations) for ideal gasses

$$\overline{G}_{T}(P) = \overline{G}_{T}^{o}(P = 1 bar) + RT \ln\left(\frac{P}{1 bar}\right)$$

$$\circ \qquad \mu_{i}(P,T) = \mu_{i}^{o}(T) + RT \ln\left(\frac{P_{i}}{1 bar}\right)$$

where
$$\underline{R} = R \times (mol)$$
 and $\overline{v_i} = v_i \times (mol^{-1})$
= 8.3144 J K⁻¹
= 0.082057 L atm K⁻¹
= 0.083144 L bar K⁻¹
 $\overline{v_i} = \frac{v_i}{mol} = [unitless]$

$$(\Delta G)_{reaction} = \Delta G^o + \underline{R}T \ln Q$$

$$\circ \quad (\Delta \mu_i)_{reaction} = \Delta \mu_i^o + \underline{R}T \ln Q$$

$$Q = \prod_i \left(\frac{P_i}{1 \ bar}\right)^{\overline{\nu}_i} \quad [or \ Q = \prod_i \left(\frac{c}{1 \ M}\right)^{\overline{\nu}_i}]$$

• Equilibrium and $\Delta G \,$ and $\Delta \mu$

•
$$\Delta G=0$$
; $\Delta \mu=0$
 $(\Delta G^{\circ})_{reaction} = -\underline{R}T \ln K_{eq}$
• $(\Delta \mu_i^{\circ})_{reaction} = -\underline{R}T \ln K_{eq}$

- $_{\circ}$ ~ interpretation of $\Delta\mu^{\circ}~$ vs $-\underline{R}T$ In K_{eq}
- How G, $\Delta G_{\text{reaction}}$, and K_{eq} vary with T and with P $_{\circ}$ $\,$ Pressure dependence

•
$$\left(\frac{\partial G}{\partial P}\right)_T = V \quad \left(\frac{\partial \Delta_{reac}G}{\partial P}\right)_T = \Delta reacV$$

• $G_T(P_2) = G_T(P_1) + \int_{P_1}^{P_2} VdP \quad \stackrel{ideal gas}{=} \quad G_T(P_1) + nRT \ln \frac{P_2}{P_1}$
• $\Delta_{reac}G_T(P_2) = \Delta_{reac}G_T(P_1) + \int_{P_1}^{P_2} \Delta_{reac}VdP$

• Temperature dependence $(2\overline{a})$

$$\left(\frac{\partial G}{\partial T}\right)_{P} = -\overline{S} \quad \overline{S} \text{ is molar entropy of pure substance}$$
$$\left(\frac{\partial \Delta G_{reaction}}{\partial T}\right)_{P} = -\Delta S_{reaction} \quad \Delta S_{reaction} \approx \sum_{i} v_{i} \left(\overline{S}_{i}\right)_{pure}$$

■ more rigorously (later)

$$\left(\frac{\partial \mu_i}{\partial T}\right)_p = -\overline{S}_i \qquad [\overline{S}_i = \left(\frac{\partial S}{\partial n_i}\right)_{T,P,n_j} = partial \ molar \ entropy]$$
$$\left(\frac{\partial \Delta \mu}{\partial T}\right)_p = -\Delta S \quad \Delta S = \sum_i v_i \overline{S}_i = \sum_i v_i \left(\frac{\partial S}{\partial n_i}\right)_{T,P,n_j}$$

$$\frac{G}{T} = \frac{H}{T} - S$$

$$\left(\frac{\partial (G/T)}{\partial T}\right)_{P} = -\frac{H}{T^{2}}$$

$$\left(\frac{\partial (\Delta G_{reac}/T)}{\partial T}\right)_{P} = -\frac{\Delta H_{reac}}{T^{2}}$$

• From
$$\frac{\left(\Delta\mu_{i}^{o}\right)_{reaction}}{T} = -\underline{R}\ln K_{eq}$$

 $\left(\frac{\partial\ln K_{eq}}{\partial T}\right)_{P} = \frac{\Delta H_{reac}}{\underline{R}T^{2}}$

• Le Chatlier's Principle

- Introduction to multicomponent systems
 - Partial molar volume as illustration of more general partial molar quantities

•
$$\overline{V}_i = \left(\frac{\partial V}{\partial n_i}\right)_{T,P,n_i \neq n_i}$$

- Conceptual interpretation of partial molar volume versus molar volume
- Total differentials for multicomponent systems ($dw_{other} = 0$)

$$U(S,V,n_{1},...,n_{N}) \qquad dU = TdS - PdV + \sum_{i=1}^{N} \left(\frac{\partial U}{\partial n_{i}}\right)_{S,V,n_{j}\neq n_{i}} dn_{i}$$

$$H(S,P,n_{1},...,n_{N}) \qquad dH = TdS + VdP + \sum_{i=1}^{N} \left(\frac{\partial H}{\partial n_{i}}\right)_{S,P,n_{j}\neq n_{i}} dn_{i}$$

$$A(T,V,n_{1},...,n_{N}) \qquad dA = -SdT - PdV + \sum_{i=1}^{N} \left(\frac{\partial A}{\partial n_{i}}\right)_{T,V,n_{j}\neq n_{i}} dn_{i}$$

$$G(T,P,n_{1},...,n_{N}) \qquad dG = -SdT + VdP + \sum_{i=1}^{N} \left(\frac{\partial G}{\partial n_{i}}\right)_{T,P,n_{j}\neq n_{i}} dn_{i}$$

• Chemical potential is partial molar Gibbs free energy

$$\overline{\boldsymbol{G}}_{i} = \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{n}_{i}}\right)_{T,P,\boldsymbol{n}_{j}\neq\boldsymbol{n}_{i}} \equiv \boldsymbol{\mu}_{i}$$

• Total extensive property is sum of partial molar properties, e. g.

$$V_{total} = \sum_{i}^{N} n_{i} \overline{V}_{i} \qquad G = \sum_{i}^{N} n_{i} \overline{G}_{i} \qquad H = \sum_{i}^{N} n_{i} \overline{H}_{i}$$

 Many thermodynamic relationships among variables for pure systems hold for partial molar quantities for each component, e.g.

$$G = H - TS \implies \overline{G}_i = \overline{H}_i - T\overline{S}_i$$

or
$$H = U + PV \implies \overline{H}_i = \overline{U}_i + P\overline{V}_i$$

o Gibbs-Duhem relationship

$$\sum_{i=1}^{N} X_{i} \left(\frac{\partial \overline{V}_{i}}{\partial n_{A}} \right)_{T,P,n_{j} \neq n_{A}} = 0$$

• Corrections for nonideality (first look)

$$\mu_i(T) = \mu_i^o(T) + RT \ln a_i$$

 $a_i = activity$

$$\circ \quad a_i = \left(\frac{\gamma_i P_i}{1 \, bar}\right) \quad or \quad a_i = \left(\frac{\gamma_i c_i}{1 \, M}\right), \quad etc.$$

where γ_i = activity coefficient

[for gases activity is same as 'fugacity']

- activity of pure liquids and solids = 1
- Example of calculation of fugacity from experimental measures or from equation of state $\lim_{t \to P} f \to P$

$$\lim_{P_i \to 0} J_i \to P_i \\
\ln\left(\frac{f_i}{P_i}\right) = \ln\left(\gamma_i\right) = \frac{1}{RT} \int_0^p \left(\overline{V_i} - \frac{RT}{P_i}\right) dP_i = \frac{1}{RT} \int_0^p \left(\overline{V_i}^{actual} - \overline{V_i}^{ideal}\right) = \int_{P_i \to 0}^p \frac{(z-1)}{P'} dP' \quad where \ z = \frac{\overline{V_{actual}}}{\overline{V_{ideal}}}$$

get $\overline{V}(P_i)$ or z from equation of state or measurement

- o Writing Q and Keq with activities and activity coefficients
- One component phase equilibria
 - $\mu_i^{\alpha} = \mu_i^{\beta} = \mu_i^{\gamma} = ...$ chemical potential of each component same in each phase
 - $\left(\frac{\partial \mu_i^{\alpha}}{\partial T}\right)_p = -\overline{S}_i^{\alpha}$ if $\mu_i^{\alpha} \neq \mu_i^{\beta}$, how the relative entropies and changes in T will lead to

phase equilibrium

- Phase rule: f=2+c-p (f=3-p for one component)
- P vs T for one-component phase equilibrium:

$$\left(\frac{dP}{dT}\right)_{phase \; equilib} = \frac{\Delta \overline{S}_{\phi}}{\Delta \overline{V}_{\phi}} = \frac{\Delta \overline{H}_{\phi}}{T\Delta \overline{V}_{\phi}}$$

- One component (P vs T) phase diagrams
 - Phases present
 - Slope of $\left(\frac{dP}{dT}\right)_{\text{phase equilib}} = \frac{\Delta \overline{S}_{\phi}}{\Delta \overline{V}_{\phi}} = \frac{\Delta \overline{H}_{\phi}}{T\Delta \overline{V}_{\phi}}$ for s $\leftrightarrow \ell$, s \leftrightarrow g, and $\ell \leftrightarrow$ g lines on phase

diagram

- Triple point
- Critical point

- Vapor pressure over pure liquids and solids
 - (s \leq g and $\ell \leq$ g), sublimation and vaporization.

$$\begin{pmatrix} \frac{d \ln P}{dT} \end{pmatrix}_{phase \ equilib} = \frac{\Delta \overline{H}_{vaporization}}{RT^2} \quad (Clausius - Clapeyron) \\ \ln \left[\frac{P_2}{P_1} \right] = \frac{1}{R} \int_{T_1}^{T_2} \frac{\Delta \overline{H}_{vaporization}}{T^2} dT \\ = -\frac{\Delta \overline{H}_{vap}}{R} \left[\frac{1}{T_2} - \frac{1}{T_1} \right] \quad if \ \Delta \overline{H}_{vaporization} \ independent \ of \ T$$

$$\begin{pmatrix} \frac{dP}{dT} \end{pmatrix}_{phase \; equilib} = \frac{\Delta \overline{H}_{fusion}}{T(\overline{V}_{\ell} - \overline{V}_{s})} P_{2} - P_{1} = \int_{T_{1}}^{T_{2}} \frac{\Delta \overline{H}_{fusion}}{T(\overline{V}_{\ell} - \overline{V}_{s})} dT P_{2} - P_{1} = \frac{\Delta \overline{H}_{fusion}}{(\overline{V}_{\ell} - \overline{V}_{s})} \ln\left[\frac{T_{2}}{T_{1}}\right] \quad if \; \Delta \overline{H}_{fusion} \; and \; \Delta V_{fusion} \; independent \; of \; T$$

- Ideal solutions
 - o Similar within and between component forces
 - $\circ \quad P_A = X_A P_A^{\bullet} \quad and \quad P_B = X_B P_B^{\bullet} \quad Raoult's \ Law$

$$\circ \quad \mu_i^{\text{soln}}(T, X_i) = \mu_i^{\ell \bullet}(T) + RT \ln X_i \qquad (ideal \ solution \Rightarrow \gamma_i = I)$$

- Thermodynamics of ideal solutions
 - $\Delta V_{mix} = \Delta H_{mix} = \Delta U_{mix} = 0$

•
$$\Delta G_{mix} = \sum_{k} n_k RT \ln X_k$$

•
$$\Delta S_{mix} = -\sum_{k} n_k R \ln X_k$$

- Multicomponent phase equilibria
 - Phase rule f=2+c-p
 - T vs X_A phase diagrams (P constant)
 - Understand the Cd-Bi diagram (handout #48, slide 4, et al; and E&R Fig 9.26))
 - Understand the benzene-toluene T vs X_{benzene} phase diagram (handout #48, slide 23, et al; and E&R Fig. 9.6) and fractional distillation
- Colligative properties $(X_{B} \text{ is mole fraction of solvent in solution})$
 - Change of solvent chemical potential upon solution formation at T and P: $\mu_B^{\text{soln}}(T, P) = \mu_B^{\ell \bullet}(T, P) + RT \ln[\gamma_B X_B]$
 - $_{\circ}$ Correction of X_B for formation of ions in dilute solutions of electrolytes.
 - Freezing point lowering:
 - Start: pure solid[•]_B \rightleftharpoons pure ℓ iquid[•]_B at T^{\bullet}_{ℓ} normal melting T_{fusion}
 - Add X_A and change T: pure solid $_{B}^{\bullet} \rightleftharpoons$ solution(X_B) at T_f
 - Change in $\Delta \mu_B$ due to solution formation: $RT \ln [\gamma_B X_B] = 0$

• Change in
$$\Delta \mu_{\rm B}$$
 due to $T^{\bullet}_{\rm f} \rightarrow T_{\rm f}$: $-\int_{T_{\rm f}}^{T_{\rm f}} \frac{\Delta \overline{H}_{fusion}}{T^2} dT$

• Net change in
$$\Delta \mu_B = 0$$

$$R \ln [\gamma_B X_B] = -\Delta \overline{H}_{fusion} \left[\frac{1}{T_f} - \frac{1}{T_f^*} \right]$$
$$\bullet \quad \gamma_B X_B = \exp \left[-\frac{\Delta \overline{H}_{fusion}}{R} \left[\frac{1}{T_f} - \frac{1}{T_f^*} \right] \right]$$
$$T_f = \frac{T_f^* \Delta \overline{H}_{fusion}}{\Delta \overline{H}_{fusion} - RT_f^* \ln (\gamma_B X_B)}$$

- Boiling point elevation:
 - Start: pure liquid[•]_B \rightleftharpoons pure vapor[•]_B at $T^{\bullet}_{h_{p}}$ normal boiling $T_{h,p}, P^{\bullet}_{B} = 1$ atm
 - Add X_A and change T: solution $(X_B) \rightleftharpoons pure vapor_B^{\bullet}$ at T_{bp} , $P_B^{\bullet} = 1$ atm
 - Change in $\Delta \mu_B$ due to solution formation: $0 RT \ln [\gamma_B X_B]$
 - Change in $\Delta \mu_{B}$ due to $T^{\bullet}_{b.p.} \rightarrow T_{b.p.} : -\int_{T_{f}^{\bullet}}^{T_{f}} \frac{\Delta \overline{H}_{vaporization}}{T^{2}} dT$
 - Net change in ΔµB =0

$$\gamma_{B}X_{B} = \exp\left[\frac{\Delta \overline{H}_{vaporization}}{R}\left[\frac{1}{T_{bp}} - \frac{1}{T_{bp}^{\bullet}}\right]\right]$$
$$T_{bp} = \frac{T_{bp}^{\bullet}\Delta \overline{H}_{vaporization}}{\Delta \overline{H}_{vaporization} + RT_{bp}^{\bullet}\ln(\gamma_{B}X_{B})}$$

- Osmotic pressure
 - Start:: pure ℓ iquid $_{B}^{\bullet}(P_{0}, left) \rightleftharpoons pure \ell$ iquid $_{B}^{\bullet}(P_{0}, right)$ at T
 - Add X_A and change P: solution $(X_{R}, P_{0} + \pi, left) \rightleftharpoons pure solvent(P_{0}, right)$
 - Change in μ_{left} due to solution formation: $RT \ln [\gamma_B X_B]$
 - Change in μ_{left} due to $P_0 \rightarrow P_0 + \pi : \pi \overline{V}_B$
 - Net change in μ_{left} =0

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$$\pi = \frac{-RT \ln[\gamma_B X_B]}{\overline{V_B}}$$
 dilute solutions $\pi \approx \frac{n_{solute} RT}{V_{solvent}}$

- o Obtaining activity coefficients from measurement of colligative properties
- Electrochemistry
 - $\circ \quad \text{Const T and P: } \Delta \mu_{\text{reaction}} \leq w_{\text{other}}$
 - For electrochemical cell:
 - $\Delta \mu_{\text{reaction}} = -n \mathcal{F} \Phi^{\dagger}$

 $\Delta \mu_{\text{reaction}} = -n \mathcal{F} \Phi$ for reversible cell ($\Phi_{\text{rev}} > \Phi^{\dagger}$)

 $(\Phi = \Phi_{cell} \text{ is electromotive force; EMF is denoted as } \mathcal{E} \text{ in many texts})$

$$\Phi = \Phi^{\circ} - \frac{RT}{nF} \ln Q \quad Nernst \ Equation$$

$$\Phi = \Phi^{\circ} - \frac{0.02569}{n} \ln Q \quad at \ T = 298.15K$$

$$\Phi^{\circ} = \frac{RT}{nF} \ln K_{eq}$$

$$\Phi^{\circ} = \frac{0.02569}{n} \ln K_{eq} \quad at \ T = 298.15K$$

- Responsible for three particular redox reactions (Handout #56, slides 4et al, 12(6)et al, 19 et al.
- o Obtaining activity coefficients from measurement of cell EMF's
- $_{\circ}$ All thermodynamic relationships for $\Delta\mu_{\text{reaction}}$ can be applied to Φ_{cell} :

$$\begin{pmatrix} \frac{\partial \Delta \mu}{\partial T} \end{pmatrix}_{P} = -\Delta \overline{S} \qquad \Rightarrow \begin{pmatrix} \frac{\partial \Phi}{\partial T} \end{pmatrix}_{P} = \frac{\Delta \overline{S}}{n\mathcal{F}} \\ \begin{pmatrix} \frac{\partial \Delta \mu}{T} \\ \frac{\partial T}{\partial T} \end{pmatrix}_{P} = \frac{-\Delta \overline{H}}{T^{2}} \qquad \Rightarrow \begin{pmatrix} \frac{\partial \Phi}{T} \\ \frac{\partial T}{\partial T} \end{pmatrix}_{P} = \frac{\Delta \overline{H}}{n\mathcal{F}T^{2}}, \text{ etc}$$

- Concluding factoids
 - Thermodynamics is useful
 - Electrical potential across membranes (e.g. neurons) can be calculated using Nernst equation
 - Non-idealities in solutions
 - Azeotropes and eutectics: constant boiling and melting solutions
 Negative deviation from Raoult's Law (stronger forces; high boiling azeotrope)
 Positive deviation from Raoult's Law (weaker forces; low boiling azeotrope)
 - Gibbs-Duhem:
 - partial molar properties for differing components are interdependent

Debye-Huckel

Theoretical method for calculating γ_{\pm} for electrolytes (note $\gamma_{\pm} \leq 1$)