## Review Topics the Entire Puorter !!!

In reviewing material one should not memorize the entries as 'plug-in' formulas. But should understand i) how they arise from the basic laws of thermodynamics; ii) what are the limits on their generality and under what special conditions they apply; and iii) how they qualitatively reflect the 'logic' of the natural world and, in some cases, the underlying properties of molecules and matter.

- Mathematical techniques (to be employed throughout course as needed)
- Exact and inexact differentials
- State functions
- Equations of state
- Thermodynamic variables (P, V, T, variables for wother ) and ideal gas thermometer to operationally measure $T$
$\mathrm{P}=\mathrm{F} / \mathrm{A}$
- System and surroundings Ideal Gas
- $P V=n R T$
- $E=(3 / 2)$ nRT (monatomic; no "internal" structure, translational kinetic energy in 3 dimensions)
- Approximation for real gasses
- Van der Waals $\left(\mathrm{P}-\frac{\mathrm{a}}{\overline{\mathrm{V}}^{2}}\right)(\overline{\mathrm{V}}-\mathrm{nb})=\mathrm{RT}$; physical meaning of a and b
- Other approximate equations of state exist (e.g. Virial power series expansion)
- Work
$w=$ force $\bullet$ displacement
$\bar{d} \mathrm{w}$ inexact differential, depends on path
$\pi \mathrm{w}=-\mathrm{Pext} d \mathrm{~V}$ (P-V work)
$\mathrm{w}=\int \pi \mathrm{w}$
$w=-P_{\text {ext }} \Delta V \quad\left(P_{\text {ext }}=\right.$ constant, $\left.w_{o t h e r}=0\right)$
- $w=-n R T \ln \left(V_{f} / V_{i}\right) \quad$ (isothermal, reversible, ideal gas, wother $=0$ )
- Heat
- $\overline{d q}$ inexact differential depends on path
$\overline{\mathrm{q}}=\mathrm{C} d T$
$\mathrm{q}=\int \mathrm{C} d T$
- First Law of Thermodynamics
- $\mathrm{U}=$ internal energy
- $d \mathrm{U}=d \mathrm{q}+\pi \mathrm{w} \quad$ (definition, for closed system)
- Equivalent statements of First Law of Thermodynamics
- U is a state function
- $d \mathbf{U}$ is an exact differential
- $\Delta \mathrm{U}_{\text {sys }}=-\Delta \mathrm{U}_{\text {surr }}$
- $\mathrm{H} \equiv \mathrm{U}+(\mathrm{PV})_{\text {internal }}$
- H is a state function
- $\Delta \mathrm{H}=\Delta \mathrm{U}+\Delta(\mathrm{PV})=\Delta \mathrm{U}+\mathrm{P}_{\mathrm{f}} \mathrm{V}_{\mathrm{f}}-\mathrm{Pi}_{\mathrm{i}} \mathrm{V}_{\mathrm{i}}$
- Differential relationships for state functions
( $\mathrm{n}_{\mathrm{i}}=$ number moles of substance i ; $\mathrm{ov}_{1}$.. оvм are "other variables" that might define the energy of the system and be involved in $\mathrm{w}_{\text {other }}$. For example
charge, position with respect gravitational attraction, etc)


more commonly (for closed system, $\mathrm{dn}_{\mathrm{i}}=0$; and no work other, $d \mathrm{w}_{\text {other }}=0$ )
- $\quad \mathrm{dU}(\mathrm{T}, \mathrm{V})=\left(\frac{\partial \mathrm{U}}{\partial \mathrm{T}}\right)_{\mathrm{V}} \mathrm{dT}+\left(\frac{\partial \mathrm{U}}{\partial \mathrm{V}}\right)_{\mathrm{T}} \mathrm{dV}$
- $\mathbf{d H}(\mathbf{T}, \mathbf{P})=\left(\frac{\partial \mathbf{H}}{\partial \mathbf{T}}\right)_{\mathbf{P}} \mathbf{d T}+\left(\frac{\partial \mathbf{H}}{\partial \mathbf{P}}\right)_{\mathrm{T}} \mathrm{dP}$
- Definitions relationships (closed system)
- $d \mathrm{U}=d \mathrm{q}-\mathrm{P}_{\text {ext }} \mathrm{dV}+d \mathrm{w}_{\text {other }}$
- $\Delta \mathrm{U}_{\mathrm{V}}=\mathrm{q}_{\mathrm{v}}+\pi \mathrm{w}_{\text {other }}$ (const V ) "heat is $\Delta \mathrm{U}$ at constant V "
- $\mathrm{n} \overline{\mathrm{C}}_{\mathrm{V}} \equiv\left(\frac{\mathrm{dq}}{\mathrm{dT}}\right)_{\mathrm{V}} \Rightarrow d \mathrm{U}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{V}} d \mathrm{~T}+\pi \mathrm{w}_{\text {other }}$ (const V )
- $\mathrm{dH}=\pi \mathrm{q}+\mathrm{V} d \mathrm{P}+\pi \mathrm{w}_{\text {other }}\left(\mathrm{P}_{\text {ext }}=\right.$ Pinternal ; thus usually for a reversible process; but OK for any infinitesimal process)
- $\Delta H \mathrm{P}=\mathrm{qp}+\mathrm{w}_{\text {other }} \quad\left(\right.$ const $\mathrm{P}_{\text {ext }} ; \mathrm{P}_{\text {intial }}=\mathrm{P}_{\text {final }}=\mathrm{Pext}$ ) "heat is $\Delta H$ at constant $P$ "
- $\mathrm{n}_{\mathrm{P}} \equiv\left(\frac{\mathrm{dq}}{\mathrm{dT}}\right)_{\mathrm{P}} \Rightarrow d \mathrm{H}_{\mathrm{P}}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{P}} d \mathrm{~T}+\pi \mathrm{w}_{\text {other }} \quad$ (const P)
- Derived consequences (closed system, $đ$ wother $=0$ )

$$
\begin{aligned}
\mathrm{dU} & =\mathrm{n} \overline{\mathrm{C}}_{\mathrm{V}} \mathrm{dT}+\left(\frac{\partial \mathrm{U}}{\partial \mathrm{~V}}\right)_{\mathrm{T}} \mathrm{dV} \\
& \quad \text { with }\left(\frac{\partial \mathrm{U}}{\partial \mathrm{~V}}\right)_{\mathrm{T}}=\left(\frac{\mathrm{dq}}{\mathrm{dV}}\right)_{\mathrm{T}}-\mathrm{P}
\end{aligned}
$$

- $d \mathrm{U}_{\mathrm{V}}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{V}} d \mathrm{~T} \quad$ (const V , any system)
but with $\left(\frac{\partial U}{\partial V}\right)_{T}=0 \quad$ for ideal gas
- $d \mathrm{U}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{v}} d \mathrm{~T} \quad$ (ideal gas, any path)

$$
\begin{aligned}
\mathrm{dH} & =\mathrm{n} \overline{\mathrm{C}}_{\mathrm{P}} \mathrm{dT}+\left(\frac{\partial \mathrm{H}}{\partial \mathrm{P}}\right)_{\mathrm{T}} \mathrm{dP} \\
& \quad \text { with }\left(\frac{\partial \mathrm{H}}{\partial \mathrm{P}}\right)_{\mathrm{T}}=\left(\frac{\mathrm{dq}}{\mathrm{dP}}\right)_{\mathrm{T}}+\mathrm{V}
\end{aligned}
$$

- $d \mathrm{HP}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{P}} d \mathrm{~T} \quad$ (const P , any system)
since for ideal gas $d \mathrm{H}=d \mathbf{U}+d(\mathrm{PV})=d \mathbf{U}+d(\mathrm{nRT})=d \mathbf{U}+\mathrm{nR} d \mathrm{~T}$
and $d \mathrm{U}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{V}} d \mathrm{~T}$
- $d \mathrm{H}_{\mathrm{P}}=\mathrm{n}\left(\overline{\mathrm{C}}_{\mathrm{V}}+\mathrm{R}\right) d \mathrm{~T}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{P}} \mathrm{dT}$ (ideal gas, any path) with $\overline{\mathrm{C}}_{\mathrm{P}}=\overline{\mathrm{C}}_{\mathrm{V}}+\mathrm{R}$ for ideal gas
- Evaluating q, w, $\Delta \mathbf{U}, \Delta \mathrm{H}$ for various processes (closed system, no wother)

Constant volume

- $w=0$
- $\mathrm{q}_{\mathrm{v}}=\Delta \mathrm{U}_{\mathrm{v}}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{v}} \Delta \mathrm{T}$
- $\Delta \mathrm{H}_{\mathrm{v}}=\Delta \mathrm{U}_{\mathrm{V}}+\mathrm{V} \Delta \mathrm{P}=\Delta \mathrm{U}_{\mathrm{V}}+\mathrm{V}\left(\mathrm{Pf}-\mathrm{P}_{\mathrm{i}}\right)$

Constant pressure

- $\mathrm{qp}_{\mathrm{p}}=\Delta \mathrm{HP}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{P}} \Delta \mathrm{T}$
- $w=-P \Delta V$
- $\Delta U p=q+w$

Ideal gasses

- $\overline{\mathrm{C}}_{\mathrm{P}}=\overline{\mathrm{C}}_{\mathrm{V}}+\mathrm{R}$
- $\overline{\mathrm{C}}_{\mathrm{V}}=\frac{3}{2} \mathbf{R}$ for monatomic ideal gas
- $\Delta \mathrm{U}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{v}} \Delta \mathrm{T}$ (any path)
- $\Delta \mathrm{H}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{P}} \Delta \mathrm{T}$ (any path)

Isothermal path

- $q=-w$
- $\Delta \mathrm{U}=0$
- $\Delta \mathrm{H}=0$
- $w=-n R T \ln \frac{V_{f}}{V_{i}}=\square n R T \ln \frac{P_{i}}{P_{f}}$ (reversible path, note - sign)
- $\mathrm{w}=-\mathrm{P}_{\text {ext }} \Delta \mathrm{V}$ (constant $\mathrm{P}_{\text {ext }}$ expansion/compression)


## Adiabatic path

- $\mathrm{q}=0$
- $\Delta U=w=n \bar{C}_{V} \Delta T$
- $\Delta \mathrm{H}=\Delta \mathrm{U}+\Delta(\mathrm{PV})=\Delta \mathrm{U}+\mathrm{P}_{\mathrm{f}} \mathrm{V}_{\mathrm{f}}-\mathrm{Pi}_{\mathrm{i}}=\mathrm{n} \overline{\mathrm{C}}_{\mathrm{P}} \Delta \mathrm{T}$
- $T_{1}^{\frac{\bar{c}_{V}}{R}} V_{1}=T_{2}^{\frac{\bar{c}_{V}}{R}} V_{2}$ (adiabatic reversible path)
- $\mathrm{P}_{1} \mathrm{~V}_{1}^{\frac{\overline{\mathrm{C}}_{\mathrm{p}}}{\overline{\mathrm{C}}_{\mathrm{V}}}}=\mathrm{P}_{2} \mathrm{~V}_{2}^{\frac{\overline{\mathrm{C}}_{\mathrm{p}}}{\overline{\mathrm{C}}_{\mathrm{V}}}}$ (adiabatic reversible path, $\mathrm{PV} \gamma=$ constant)
- $\frac{T_{1}^{\frac{\bar{C}_{P}}{R}}}{P_{1}}=\frac{T_{2}^{\frac{\bar{C}_{P}}{R}}}{P_{2}}$ (adiabatic reversible path)


## - Thermochemistry

- $\Delta \mathrm{H}_{\text {reaction }}=\left(\mathrm{H}_{\text {products }}-\mathrm{H}_{\text {reactants }}\right)$ (vs $\Delta H$ for physical change)
- $\Delta H p=q p$
- standard states
- definition of $\bar{H}_{f}^{o} \equiv \Delta \bar{H}_{f}^{o}$
- $\Delta H^{o}{ }_{\text {reaction }}=\sum_{i} v_{i} \bar{H}_{f}^{o} \quad$ where $v_{i}$ are stoichiometric coefficients
- Hess's Law
- $\Delta \mathrm{H}_{\text {reaction }}=\Delta \mathrm{U}_{\text {reaction }}+\Delta \mathrm{n}_{\text {gas }} R T$
- $\Delta H\left(T_{2}\right)=\Delta H\left(T_{1}\right)+\int_{T_{1}}^{T_{2}} \Delta C_{P} d T$
- $\Delta \mathrm{H}$ from bond enthalpies
- Heats of reactions in ionic solution
- Statements of the Second Law of Thermodynamics
- Macroscopic properties of an isolated system eventually assume constant values (e.g. pressure in two bulbs of gas_becomes constant; two block of metal reach same T) [Andrews. p37]
- It is impossible to construct a device that operates in cycles and that converts heat into work without producing some other change in the surroundings. Kelvin's Statement [Raff p 157]; Carnot Cycle
- It is impossible to have a natural process which produces no other effect than absorption of heat from a colder body and discharge of heat to a warmer body. Clausius's Statement, refrigerator
- In the neighborhood of any prescribed initial state there are states which cannot be reached by any adiabatic process
~ Caratheodory's statement [Andrews p. 58]
- Important statements regarding entropy and statistical disorder

Greater number of microstates, greater disorder

- Each allowed microstate is equally probable
- the overwhelming number of microstates correspond to macrostates with almost identical macroscopic properties
- W, the number of microstates corresponding to the macrostate, is a measure of the DISORDER of the system in that macrostate
- A system "meanders" through all available microstates; but you are only likely to observe it in one of the overwhelming number that correspond to the equilibrium macrostate
- W, the number of microstates corresponding to the macrostate, depends on the populations of various energy levels
- Adiabatic, reversible, work changes the energy of system but not the populations of the energy levels (the disorder, W)
- Reversible heat transfers change the populations of the energy levels (and thus W)
- Carnot engines (reversible, $\mathrm{q} H>0$ at $\mathrm{T}_{\mathrm{H}} ; \mathrm{qL}<0$ at $\mathrm{T}_{\mathrm{L}} ; \mathrm{w}_{t}<0$ )
- Demonstration of machine consistent with $2^{\text {nd }}$ Law (i.e. in a cyclic process, heat can be converted to work if and only if heat is returned to surroundings at a lower temperature.
- $\varepsilon=\frac{-w_{t}}{q_{H}}=1-\frac{T_{L}}{T_{H}}$
- Reverse process corresponds to refrigerators and heat pumps
- $\sum \frac{q}{T}=\frac{q_{H}}{T_{H}}+\frac{q_{L}}{T_{L}}=0$
- $\varepsilon$ of any Carnot engine ('any working substance") has the same dependence on $T_{H}$ and $T_{L}$
- Any reversible cyclic process can be describes as a sum of (infinitesimal) Carnot cycles, thus the properties of any cyclic process are combinations of those of Carnot cycles
- Thermal properties of entropy and entropy calculations
- $d S=\frac{\vec{d} q_{r e v}}{T} ; \quad \Delta S=\int \frac{\vec{d} q_{r e v}}{T} ; \quad \oint \frac{\vec{d} q_{r e v}}{T}=0$
$\Delta S \geq \int \frac{\vec{d} q}{T} ; \quad 0 \geq \oint \frac{\vec{d} q}{T} ; \quad(=$ for reversible process; $>$ for spontaneous ['real'] process)
$\Delta S_{\text {total }=\text { universe }}=\Delta S_{\text {system }}+\Delta S_{\text {surroundings }} \geq 0$
- $S$ is a state function; $d S$ is an exact differential
- Dependence of $S$ on
- T: $\left(\frac{\partial \bar{S}}{\partial T}\right)_{V}=\frac{\bar{C}_{V}}{T} ;\left(\frac{\partial \bar{S}}{\partial T}\right)_{P}=\frac{\bar{C}_{P}}{T}$
- $\mathrm{P}:\left(\frac{\partial \bar{S}}{\partial P}\right)_{T}=-\left(\frac{\partial \bar{V}}{\partial T}\right)_{P}$
- $\mathrm{V}:\left(\frac{\partial \bar{S}}{\partial \bar{V}}\right)_{T}=\left(\frac{\partial P}{\partial T}\right)_{\bar{V}}$
- Phase: $\Delta S=\frac{\Delta H_{\text {equilibrium phase change }}}{T_{\text {equilibrium phase change }}}$
- Calculation of entropy changes for changes in P, V, T, phase
- Third Law and calculations using Third Law Entropies: $\bar{S}^{o}(T)$
- $\Delta S_{\text {reaction }}^{0}(T)=\sum_{i} v_{i} \bar{S}_{i}^{0}(T)$
- Entropy of mixing: $\Delta S=-n_{\text {total }} R \sum_{i} X_{i} \ln X_{i} \quad$ where $X_{i}=\frac{n_{i}}{n_{\text {total }}}$
- Qualitative assessment of factors affecting $\bar{S}(T, P, V)$ and $\Delta S_{\text {reaction }}$
- Derivation of various thermodynamic relationships
- mathematical tools
- properties of exact differentials
- "dividing through" by 'd?"
- Euler-Maxwell relationships
- state functions
- $U=q+w$
- $\mathrm{H} \equiv \mathrm{U}+\mathrm{PV}$
- $A \equiv U-T S$
- $\mathrm{G}=\mathrm{H}-\mathrm{TS}$
- and their total differentials (no work other, closed systems)
- $T d S=\pi q \quad T d S=\pi q_{\text {rev }} \quad\left(P_{\text {ext }}\right)_{\text {rev }}=P_{\text {int }}=P$
- $\quad d U=T d S ~-~ P d V ~ U(S, V) \quad$ internal energy
- $\mathrm{dH}=\mathrm{TdS}+\mathrm{VdP} \mathrm{H}(\mathrm{S}, \mathrm{P})$ enthalpy
- $d A=-S d T-P d V \quad A(T, V) \quad$ Helmholtz free energy
- $\quad d G=-S d T+V d P \quad G(T, P) \quad$ Gibbs free energy
- Free energy defined

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- A\equivU - TS
- G=H - TS
- dA= - SdT - PdV
- dG= - SdT +VdP
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- Free energy: equilibrium (=) and spontaneity (<)
- $\Delta \mathrm{A}_{T, V} \leq 0$
- $\Delta \mathrm{G}_{\mathrm{T}, \mathrm{P}} \leq 0$
- Relationship of $\Delta \mathrm{G}_{\mathrm{T}, \mathrm{P}} \leq 0$ to $2^{\text {nd }}$ Law

$$
\begin{aligned}
-\frac{\Delta G_{T, P}}{T} & =-\frac{\Delta H}{T}+\Delta S \geq 0 \\
\Delta S_{\text {surr }} & \geq-\frac{\left(\Delta H_{P}\right)_{s y s}}{T} \text { disorders surroundings }
\end{aligned}
$$

- $\Delta S_{s y s}$ disorders system
$\Delta S_{\text {sur }}+\Delta S_{\text {sys }} \geq 0$
- Calculation of $\Delta \mathrm{G}$ for physical processes (expansion of gasses, phase changes, etc.)
- Calculation of $\Delta G_{\text {reaction }}$ from $\Delta G_{f}$ or $\Delta H_{f}$ and $\Delta S$ or reactants and products
- $\quad \Delta \mathrm{G}_{\mathrm{T}, \mathrm{P}}$ at arbitrary partial pressures (concentrations) for ideal gasses

$$
\begin{aligned}
& \bar{G}_{T}(P)=\bar{G}_{T}^{o}(P=1 \text { bar })+R T \ln \left(\frac{P}{1 \text { bar }}\right) \\
& \mu_{i}(P, T)=\mu_{i}^{o}(T)+R T \ln \left(\frac{P_{i}}{1 \text { bar }}\right)
\end{aligned}
$$

$$
\begin{aligned}
\text { where } \underline{R} & =R \times(\mathrm{mol}) \text { and } \bar{v}_{i}=v_{i} \times\left(\mathrm{mol}^{-1}\right) \\
& =8.3144 \mathrm{~J} \mathrm{~K}^{-1} \\
& =0.082057 \mathrm{Latm} \mathrm{~K}^{-1} \\
& =0.083144 \mathrm{Lbar} \mathrm{~K}^{-1} \\
\bar{v}_{i}=\frac{v_{i}}{m o l} & =[\text { unitless }]
\end{aligned}
$$

$$
\begin{aligned}
& (\Delta G)_{\text {reaction }}=\Delta G^{o}+\underline{R} T \ln Q \\
& \circ \quad\left(\Delta \mu_{i}\right)_{\text {reaction }}=\Delta \mu_{i}^{o}+\underline{R} T \ln Q \\
& Q=\prod_{i}\left(\frac{P_{i}}{1 \text { bar }}\right)^{\overline{v_{i}}} \quad\left[\text { or } Q=\prod_{i}\left(\frac{c}{1 M}\right)^{\overline{v_{i}}}\right]
\end{aligned}
$$

- Equilibrium and $\Delta \mathrm{G}$ and $\Delta \mu$
- $\Delta \mathrm{G}=0 ; \Delta \mu=0$
$\left(\Delta G^{o}\right)_{\text {reaction }}=-\underline{R} T \ln K_{e q}$ $\left(\Delta \mu_{i}^{o}\right)_{\text {reaction }}=-\underline{R} T \ln K_{\text {eq }}$
- interpretation of $\Delta \mu^{\circ}$ vs $-\underline{\mathrm{R}} \operatorname{In} \mathrm{K}_{\text {eq }}$
- How $\mathrm{G}, \Delta \mathrm{G}_{\text {reaction }}$, and $\mathrm{K}_{\text {eq }}$ vary with T and with P
- Pressure dependence
- $\left(\frac{\partial G}{\partial P}\right)_{T}=V \quad\left(\frac{\partial \Delta_{\text {reac }} G}{\partial P}\right)_{T}=\Delta$ reac $V$
- $G_{T}\left(P_{2}\right)=G_{T}\left(P_{1}\right)+\int_{P_{1}}^{P_{2}} V d P \stackrel{\text { ideal gas }}{=} G_{T}\left(P_{1}\right)+n R T \ln \frac{P_{2}}{P_{1}}$
- $\Delta_{\text {reac }} G_{T}\left(P_{2}\right)=\Delta_{\text {reac }} G_{T}\left(P_{1}\right)+\int_{P_{1}}^{P_{2}} \Delta_{\text {reac }} V d P$
- Temperature dependence

$$
\begin{aligned}
& \left(\frac{\partial \bar{G}}{\partial T}\right)_{P}=-\bar{S} \quad \bar{S} \text { is molar entropy of pure substance } \\
& \left(\frac{\partial \Delta G_{\text {reaction }}}{\partial T}\right)_{P}=-\Delta S_{\text {reaction }} \quad \Delta S_{\text {reaction }} \approx \sum_{i} v_{i}\left(\bar{S}_{i}\right)_{\text {pure }}
\end{aligned}
$$

- more rigorously (later)

$$
\begin{aligned}
& \left(\frac{\partial \mu_{i}}{\partial T}\right)_{P}=-\bar{S}_{i} \quad\left[\bar{S}_{i}=\left(\frac{\partial S}{\partial n_{i}}\right)_{T, P, n_{j}}=\text { partial molar entropy }\right] \\
& \left(\frac{\partial \Delta \mu}{\partial T}\right)_{P}=-\Delta S \quad \Delta S=\sum_{i} v_{i} \bar{S}_{i}=\sum_{i} v_{i}\left(\frac{\partial S}{\partial n_{i}}\right)_{T, P, n_{j}} \\
& \frac{G}{T}=\frac{H}{T}-S \\
& -\left(\frac{\partial(G / T)}{\partial T}\right)_{P}=-\frac{H}{T^{2}} \\
& \left(\frac{\partial\left(\Delta G_{\text {reac }} / T\right)}{\partial T}\right)_{P}=-\frac{\Delta H_{\text {reac }}}{T^{2}}
\end{aligned}
$$

- From $\frac{\left(\Delta \mu_{i}^{o}\right)_{\text {reaction }}}{T}=-\underline{R} \ln K_{\text {eq }}$

$$
\left(\frac{\partial \ln K_{e q}}{\partial T}\right)_{P}=\frac{\Delta H_{\text {reac }}}{\underline{R} T^{2}}
$$

## - Le Chatlier's Principle

- Introduction to multicomponent systems
- Partial molar volume as illustration of more general partial molar quantities
- $\bar{V}_{i}=\left(\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{n}_{\boldsymbol{i}}}\right)_{T, P, n_{j} \neq \boldsymbol{n}_{i}}$
- Conceptual interpretation of partial molar volume versus molar volume
- Total differentials for multicomponent systems $\left(\boldsymbol{d} \boldsymbol{w}_{\text {other }}=0\right)$

$$
\begin{aligned}
& \boldsymbol{U}\left(\boldsymbol{S}, \boldsymbol{V}, \boldsymbol{n}_{1}, \ldots, \boldsymbol{n}_{N}\right) \quad \boldsymbol{d} \boldsymbol{U}=\boldsymbol{T} \boldsymbol{d} \boldsymbol{S}-\boldsymbol{P d} \boldsymbol{V} \boldsymbol{+}+\sum_{i=1}^{\boldsymbol{N}}\left(\frac{\partial \boldsymbol{U}}{\partial \boldsymbol{n}_{\boldsymbol{i}}}\right)_{S, V, n_{j} \neq \boldsymbol{n}_{i}} \boldsymbol{d} \boldsymbol{n}_{\boldsymbol{i}} \\
& \boldsymbol{H}\left(\boldsymbol{S}, \boldsymbol{P}, \boldsymbol{n}_{1}, \ldots, \boldsymbol{n}_{N}\right) \quad \boldsymbol{d} \boldsymbol{H}=\boldsymbol{T} \boldsymbol{d} \boldsymbol{S}+\boldsymbol{V} d \boldsymbol{P}+\sum_{i=1}^{N}\left(\frac{\partial \boldsymbol{H}}{\partial \boldsymbol{n}_{i}}\right)_{S, P, n_{j} \neq n_{i}} \boldsymbol{d} \boldsymbol{n}_{i} \\
& \boldsymbol{A}\left(\boldsymbol{T}, \boldsymbol{V}, \boldsymbol{n}_{1}, \ldots ., \boldsymbol{n}_{N}\right) \quad \boldsymbol{d} \boldsymbol{A}=-\boldsymbol{S} d \boldsymbol{T}-\boldsymbol{P d V}+\sum_{i=1}^{N}\left(\frac{\partial A}{\partial n_{i}}\right)_{T, V, n_{j} \neq n_{i}} d \boldsymbol{n}_{i} \\
& \boldsymbol{G}\left(\boldsymbol{T}, \boldsymbol{P}, \boldsymbol{n}_{1}, \ldots ., \boldsymbol{n}_{N}\right) \quad \boldsymbol{d} \boldsymbol{G}=-\boldsymbol{S} \boldsymbol{d} \boldsymbol{T}+\boldsymbol{V} \boldsymbol{d} \boldsymbol{P}+\sum_{i=1}^{N}\left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{n}_{\boldsymbol{i}}}\right)_{T, P, \boldsymbol{n}_{j} \neq \boldsymbol{n}_{i}} \boldsymbol{d} \boldsymbol{n}_{\boldsymbol{i}}
\end{aligned}
$$

- Chemical potential is partial molar Gibbs free energy

$$
\overline{\boldsymbol{G}}_{i}=\left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{n}_{i}}\right)_{T, P, \boldsymbol{n}_{j} \neq \boldsymbol{n}_{i}} \equiv \mu_{i}
$$

- Total extensive property is sum of partial molar properties, e. g.

$$
V_{\text {total }}=\sum_{i}^{N} n_{i} \bar{V}_{i} \quad G=\sum_{i}^{N} n_{i} \bar{G}_{i} \quad H=\sum_{i}^{N} n_{i} \overline{\boldsymbol{H}}_{i}
$$

- Many thermodynamic relationships among variables for pure systems hold for partial molar quantities for each component, e.g.

$$
\begin{array}{rll}
\boldsymbol{G} \equiv \boldsymbol{H}-\boldsymbol{T S} & \Rightarrow & \bar{G}_{i}=\overline{\boldsymbol{H}}_{i}-\boldsymbol{T} \bar{S}_{i} \\
& \text { or } & \\
\boldsymbol{H} \equiv \boldsymbol{U}+\boldsymbol{P V} & \Rightarrow & \bar{H}_{i}=\bar{U}_{i}+\boldsymbol{P} \bar{V}_{i}
\end{array}
$$

- Gibbs-Duhem relationship

$$
\sum_{i=1}^{N} \boldsymbol{X}_{i}\left(\frac{\partial \overline{\boldsymbol{V}}_{i}}{\partial \boldsymbol{n}_{A}}\right)_{T, P, n_{j} \neq \boldsymbol{n}_{A}}=0
$$

- Corrections for nonideality (first look)
$\mu_{i}(T)=\mu_{i}^{o}(T)+R T \ln a_{i}$
$a_{i}=$ activity
- $a_{i}=\left(\frac{\gamma_{i} P_{i}}{1 b a r}\right)$ or $a_{i}=\left(\frac{\gamma_{i} c_{i}}{1 M}\right)$, etc.
where $\gamma_{i}=$ activity coefficient
[for gases activity is same as ' fugacity']
- activity of pure liquids and solids = 1
- Example of calculation of fugacity from experimental measures or from equation of state
$\lim _{P_{i} \rightarrow 0} f_{i} \rightarrow P_{i}$
$\ln \left(\frac{f_{i}}{P_{i}}\right)=\ln \left(\gamma_{i}\right)=\frac{1}{R T} \int_{0}^{P}\left(\bar{V}_{i}-\frac{R T}{P_{i}}\right) d P_{i}=\frac{1}{R T} \int_{0}^{P}\left(\bar{V}_{i}^{\text {actual }}-\bar{V}_{i}^{\text {ideal }}\right)=\int_{P_{1} \rightarrow 0}^{P} \frac{(z-1)}{P^{\prime}} d P^{\prime} \quad$ where $z=\frac{\bar{V}_{\text {actual }}}{\bar{V}_{\text {ideal }}}$
get $\bar{V}\left(P_{i}\right)$ or z from equation of state or measurement
- Writing Q and Keq with activities and activity coefficients
- One component phase equilibria
- $\mu_{i}^{\alpha}=\mu_{i}^{\beta}=\mu_{i}^{\gamma}=\ldots$ chemical potential of each component same in each phase
- $\left(\frac{\partial \mu_{i}^{\alpha}}{\partial T}\right)_{P}=-\bar{S}_{i}^{\alpha}$ if $\mu_{i}^{\alpha} \neq \mu_{i}^{\beta}$, how the relative entropies and changes in $T$ will lead to
phase equilibrium
- Phase rule: $f=2+c-p$ ( $f=3-p$ for one component)
- P vs T for one-component phase equilibrium:
$\left(\frac{d P}{d T}\right)_{\text {phase equilib }}=\frac{\Delta \bar{S}_{\phi}}{\Delta \bar{V}_{\phi}}=\frac{\Delta \bar{H}_{\phi}}{T \Delta \bar{V}_{\phi}}$
- One component (P vs T) phase diagrams
- Phases present
- Slope of $\left(\frac{d P}{d T}\right)_{\text {phase equilib }}=\frac{\Delta \bar{S}_{\phi}}{\Delta \bar{V}_{\phi}}=\frac{\Delta \bar{H}_{\phi}}{T \Delta \bar{V}_{\phi}}$ for $\mathrm{s} \leftrightarrow \ell, \mathrm{s} \leftrightarrow \mathrm{g}$, and $\ell \leftrightarrow \mathrm{g}$ lines on phase diagram
- Triple point
- Critical point
- Vapor pressure over pure liquids and solids
- (s $\leftrightarrows \mathrm{g}$ and $\ell \leftrightarrows \mathrm{g}$ ), sublimation and vaporization.

$$
\begin{aligned}
\left(\frac{d \ln P}{d T}\right)_{\text {phase equilib }} & \left.=\frac{\Delta \bar{H}_{\text {vaporization }}}{R T^{2}} \text { (Clausius }- \text { Clapeyron }\right) \\
\ln \left[\frac{P_{2}}{P_{1}}\right] & =\frac{1}{R} \int_{T_{1}}^{T_{2}} \frac{\Delta \bar{H}_{\text {vaporization }}}{T^{2}} d T \\
& =-\frac{\Delta \bar{H}_{v a p}}{R}\left[\frac{1}{T_{2}}-\frac{1}{T_{1}}\right] \text { if } \Delta \bar{H}_{\text {vaporization }} \text { independent of } T
\end{aligned}
$$

- Solid $\leftrightarrows$ liquid equilibrium (fusion/melting)

$$
\begin{aligned}
\left(\frac{d P}{d T}\right)_{\text {phase equilib }} & =\frac{\Delta \bar{H}_{\text {fusion }}}{T\left(\bar{V}_{\ell}-\bar{V}_{s}\right)} \\
P_{2}-P_{1} & =\int_{T_{1}}^{T_{2}} \frac{\Delta \bar{H}_{\text {fision }}}{T\left(\bar{V}_{\ell}-\bar{V}_{s}\right)} d T \\
P_{2}-P_{1} & =\frac{\Delta \bar{H}_{\text {fusion }}}{\left(\bar{V}_{\ell}-\bar{V}_{s}\right)} \ln \left[\frac{T_{2}}{T_{1}}\right] \text { if } \Delta \bar{H}_{\text {fision }} \text { and } \Delta V_{\text {fusion }} \text { independent of } T
\end{aligned}
$$

- Ideal solutions
- Similar within and between component forces
$P_{A}=X_{A} \boldsymbol{P}_{A}^{\bullet} \quad$ and $\quad P_{B}=X_{B} P_{B}^{\cdot} \quad$ Raoult's Law
- $\mu_{i}^{\text {soln }}\left(T, X_{i}\right)=\mu_{i}^{\ell \bullet}(T)+R T \ln X_{i} \quad$ (ideal solution $\left.\Rightarrow \gamma_{i}=1\right)$
- Thermodynamics of ideal solutions
- $\Delta \mathrm{V}_{\text {mix }}=\Delta \mathrm{H}_{\text {mix }}=\Delta \mathrm{U}_{\text {mix }}=0$
- $\Delta G_{\text {mix }}=\sum_{k} n_{k} R T \ln X_{k}$
- $\Delta S_{\text {mix }}=-\sum_{k} n_{k} R \ln X_{k}$
- Multicomponent phase equilibria
- Phase rule f=2+c-p
- T vs $\mathrm{X}_{\mathrm{A}}$ phase diagrams (P constant)
- Understand the Cd-Bi diagram (handout \#48, slide 4, et al; and E\&R Fig 9.26))
- Understand the benzene-toluene T vs X ${ }_{\text {benzene }}$ phase diagram (handout \#48, slide 23, et al; and E\&R Fig. 9.6) and fractional distillation
- Colligative properties ( $X_{B}$ is mole fraction of solvent in solution)
- Change of solvent chemical potential upon solution formation at T and P: $\mu_{B}^{s o \ln }(T, P)=\mu_{B}^{\ell \bullet}(T, P)+R T \ln \left[\gamma_{B} X_{B}\right]$
- Correction of $X_{B}$ for formation of ions in dilute solutions of electrolytes.
- Freezing point lowering:
- Start: pure solid ${ }_{B}^{*} \rightleftarrows$ pure liquid ${ }_{B}^{*}$ at $T_{f}^{*} \quad$ normal melting $T_{\text {fusion }}$
- Add $\mathrm{X}_{\mathrm{A}}$ and change T : pure solid ${ }^{*} \rightleftarrows \operatorname{solution}\left(X_{B}\right)$ at $T_{f}$
- Change in $\Delta \mu_{\mathrm{B}}$ due to solution formation: $R T \ln \left[\gamma_{B} X_{B}\right]-0$
- Change in $\Delta \mu_{\mathrm{B}}$ due to $\mathrm{T}_{\mathrm{f}} \rightarrow \mathrm{T}_{\mathrm{f}}:-\int_{T_{f}^{*}}^{T_{f}} \frac{\Delta \bar{H}_{\text {fision }}}{T^{2}} d T$
- Net change in $\Delta \mu_{\mathrm{B}}=0$

$$
R \ln \left[\gamma_{B} X_{B}\right]=-\Delta \bar{H}_{\text {fusion }}\left[\frac{1}{T_{f}}-\frac{1}{T_{f}^{\dot{f}}}\right]
$$

- $\gamma_{B} X_{B}=\exp \left[-\frac{\Delta \bar{H}_{\text {fusion }}}{R}\left[\frac{1}{T_{f}}-\frac{1}{T_{f}^{\cdot}}\right]\right]$

$$
T_{f}=\frac{T_{f}^{*} \Delta \bar{H}_{\text {fusion }}}{\Delta \bar{H}_{\text {fusion }}-R T_{f}^{*} \ln \left(\gamma_{B} X_{B}\right)}
$$

- Boiling point elevation:
- Start: pure liquid ${ }_{B}^{*} \rightleftarrows$ pure vapor ${ }_{B}^{\bullet}$ at $T_{b, p .}^{*} \quad$ normal boiling $T_{b, p}, P_{B}^{*}=1 \mathrm{~atm}$
- Add $X_{A}$ and change T : solution $\left(X_{B}\right) \rightleftarrows$ pure vapor ${ }_{B}^{\bullet}$ at $T_{b p}, P_{B}^{\bullet}=1 \mathrm{~atm}$
- Change in $\Delta \mu_{\mathrm{B}}$ due to solution formation: $0-R T \ln \left[\gamma_{B} X_{B}\right]$
- Change in $\Delta \mu_{\mathrm{B}}$ due to $\mathrm{T}_{\text {b.p. }} \rightarrow \mathrm{T}_{\text {b.p. }}:-\int_{T_{f}^{*}}^{T_{f}} \frac{\Delta \bar{H}_{\text {vaporization }}}{T^{2}} d T$
- Net change in $\Delta \mu_{\mathrm{B}}=0$
- $\gamma_{B} X_{B}=\exp \left[\frac{\Delta \bar{H}_{\text {vaporization }}}{R}\left[\frac{1}{T_{b p}}-\frac{1}{T_{b p}^{*}}\right]\right]$

$$
T_{b p}=\frac{T_{b p}^{*} \Delta \bar{H}_{v a p o r i z a t i o n ~}}{\Delta \bar{H}_{\text {vaporization }}+R T_{b p}^{*} \ln \left(\gamma_{B} X_{B}\right)}
$$

- Osmotic pressure
- Start:: pure liquid ${ }_{B}^{*}\left(P_{0}\right.$, left $) \rightleftarrows$ pure $\ell$ iquid ${ }_{B}^{*}\left(P_{0}\right.$, right $)$ at $T$
- Add $\mathrm{X}_{\mathrm{A}}$ and change P : solution $\left(X_{B}, P_{0}+\pi\right.$, left $) \rightleftarrows$ pure solvent $\left(P_{0}\right.$, right $)$
- Change in $\mu_{\text {left }}$ due to solution formation: $R T \ln \left[\gamma_{B} X_{B}\right]$
- Change in $\mu_{\text {left }}$ due to $\mathrm{P}_{0} \rightarrow \mathrm{P}_{0}+\pi: \pi \bar{V}_{B}$
- Net change in $\mu_{\text {left }}=0$
- $\pi=\frac{-R T \ln \left[\gamma_{B} X_{B}\right]}{\bar{V}_{B}}$ dilute solutions $\pi \approx \frac{n_{\text {solute }} R T}{V_{\text {solvent }}}$
- Obtaining activity coefficients from measurement of colligative properties
- Electrochemistry
- Const T and P: $\Delta \mu_{\text {reaction }} \leq W_{\text {other }}$
- For electrochemical cell:
$\Delta \mu_{\text {reaction }}=-\mathrm{n} \mathcal{F} \Phi^{t}$
$\Delta \mu_{\text {reaction }}=-\mathrm{n} \mathcal{F} \Phi$ for reversible cell ( $\Phi_{\text {rev }}>\Phi^{\dagger}$ )
( $\Phi \equiv \Phi_{\text {cell }}$ is electromotive force; EMF is denoted as $\mathcal{E}$ in many texts)
$\Phi=\Phi^{\circ}-\frac{R T}{n \mathcal{F}} \ln Q \quad$ Nernst Equation
$\circ$
$\Phi=\Phi^{\circ}-\frac{0.02569}{n} \ln Q \quad$ at $T=298.15 K$
$\Phi^{\circ}=\frac{R T}{n \mathcal{F}} \ln K_{\text {eq }}$
$\Phi^{\circ}=\frac{0.02569}{n} \ln K_{\text {eq }} \quad$ at $T=298.15 \mathrm{~K}$
- Responsible for three particular redox reactions (Handout \#56, slides 4et al, 12(6)et al, 19 et al.
- Obtaining activity coefficients from measurement of cell EMF's
- All thermodynamic relationships for $\Delta \mu_{\text {reaction }}$ can be applied to $\Phi_{\text {cell }}$ :

$$
\begin{aligned}
& \left(\frac{\partial \Delta \mu}{\partial T}\right)_{P}=-\Delta \bar{S} \quad \Rightarrow\left(\frac{\partial \Phi}{\partial T}\right)_{P}=\frac{\Delta \bar{S}}{n \mathcal{F}} \\
& \left(\frac{\partial \frac{\Delta \mu}{T}}{\partial T}\right)_{P}=\frac{-\Delta \bar{H}}{T^{2}} \quad \Rightarrow\left(\frac{\partial \frac{\Phi}{T}}{\partial T}\right)_{P}=\frac{\Delta \bar{H}}{n \mathcal{F} T^{2}}, \text { etc }
\end{aligned}
$$

- Concluding factoids
- Thermodynamics is useful
- Electrical potential across membranes (e.g. neurons) can be calculated using Nernst equation
- Non-idealities in solutions
- Azeotropes and eutectics: constant boiling and melting solutions Negative deviation from Raoult's Law (stronger forces; high boiling azeotrope) Positive deviation from Raoult's Law (weaker forces; low boiling azeotrope)
- Gibbs-Duhem:
partial molar properties for differing components are interdependent
Debye-Huckel
Theoretical method for calculating $\gamma_{ \pm}$for electrolytes (note $\gamma_{ \pm} \leq 1$ )

