

Lecture 10  
Chemistry 163B  
Refrigerators and  
Generalization of  
Ideal Gas Carnot  
(four steps to exactitude)

E&R<sub>4th</sub> pp 125-129, 132-139 Raff pp. 159-164  
E&R<sub>3rd</sub> pp 86-91, 109-111

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*statements of the Second Law of Thermodynamics*

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1. Macroscopic properties of an isolated system eventually assume constant values (e.g. pressure in two bulbs of gas becomes constant; two block of metal reach same T) [*Andrews. p37*]
2. It is impossible to construct a device that operates in cycles and that converts heat into work without producing some other change in the surroundings. *Kelvin's Statement* [*Raff p 157*]; *Carnot Cycle*
3. It is impossible to have a natural process which produces no other effect than absorption of heat from a colder body and discharge of heat to a warmer body. *Clausius's Statement, refrigerator*
4. In the neighborhood of any prescribed initial state there are states which cannot be reached by any adiabatic process  
~ *Caratheodory's statement* [*Andrews p. 58*]

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*perpetual motion and the 1<sup>st</sup> and 2<sup>nd</sup> Laws of thermodynamics*



**cyclic machines:**

**1<sup>st</sup> Law**  $\Delta U=0$   $q=-w_{\text{sys}}=w_{\text{surr}}$



**2<sup>nd</sup> Law**  $q_{\text{in}} > w_{\text{surr}}$



It is impossible to construct a device that operates in cycles and that converts heat into work without producing some other change in the surroundings.  
*Kelvin's Statement [Raff p 157]; Carnot Cycle*

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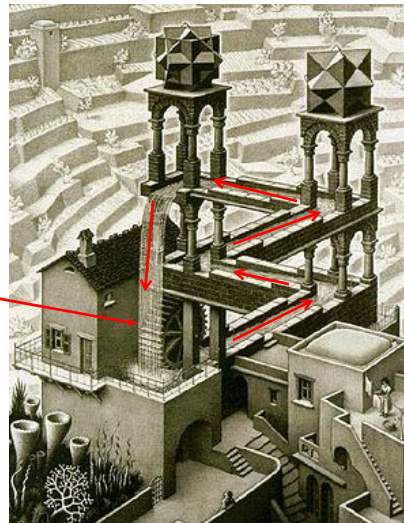
*perpetual motion of the **first kind** (produce work with no heat input)  
Escher "Waterfall"*

**cyclic machine**  
(water 'flows downhill'  
to pillar on left;  
'round and 'round)

**no energy input**

**work on surroundings**

**VIOLATES 1<sup>st</sup> LAW**



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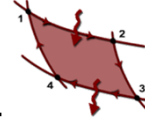
*remember Carnot engine*

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- does net work on surroundings

**BUT**

- extracts heat from surroundings at  $T_U$   
 $q_{U(I)} < 0$
- gives off heat to surroundings at  $T_L$   
 $q_{L(III)} > 0$

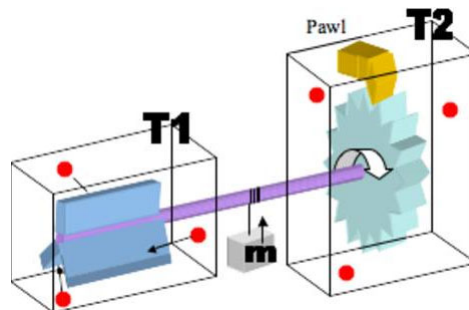


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*perpetual motion of the **second kind** (produce work extracting heat from cooler source to run machine at warmer temperature)*

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### Brownian Ratchet



get work only if  $T_1 > T_2$

[https://en.wikipedia.org/wiki/Brownian\\_ratchet.](https://en.wikipedia.org/wiki/Brownian_ratchet)

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*roadmap for second law*

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- ✓ 1. Phenomenological statements (what is ALWAYS observed)
- ✓ 2. Ideal gas Carnot [*reversible*] cycle efficiency of heat → work (Carnot cycle transfers heat only at  $T_U$  and  $T_L$ )
- 3. Any cyclic engine operating between  $T_U$  and  $T_L$  must have an equal or lower efficiency than Carnot OR VIOLATE one of the phenomenological statements (observations)
- 4. Generalize Carnot to any reversible cycle (E&R fig 5.4)
- 5. Show that for this REVERSIBLE cycle  
 $q_U + q_L \neq 0$  ( $\bar{d}q$  inexact differential)  
*but*  
 $\frac{q_U}{T_U} + \frac{q_L}{T_L} = 0$  (something special about  $\frac{\bar{d}q_{rev}}{T}$ )
- 6. S, entropy and spontaneous changes

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*goals for lecture[s]*

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- Carnot in reverse: refrigerators and heat pumps
- Show that  $\epsilon_{\text{ideal gas rev Carnot}} \geq \epsilon_{\text{any other machine}}$  *otherwise one of the phenomenological statements violated*
- Not only for ideal gas but  $\epsilon_{\text{ideal gas rev Carnot}} = \epsilon_{\text{any other rev Carnot}}$
- $dS \equiv \frac{\bar{d}q_{reversible}}{T}$      $\oint dS = \oint \frac{\bar{d}q_{reversible}}{T} = 0$   
 and S is **STATE FUNCTION**

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*remember Carnot engine*

---

• does net work on surroundings

and

• net heat  $q_{U(I)} + q_{L(III)} = -w_{\text{total}}$

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*Carnot engine arithmetic (for below table see handout) ➡*

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ENGINE	q	W <sub>sys</sub>	W <sub>surr</sub>	
I. isothermal expansion	$+nRT_U \ln \frac{P_1}{P_2}$ 1.3	$-nRT_U \ln \frac{P_1}{P_2}$ 1.2	$+nRT_U \ln \frac{P_1}{P_2}$	heat in at T <sub>H</sub> work out
II adiabatic expansion	0	$n\bar{C}_V(T_U - T_U)$ 2.4	$-n\bar{C}_V(T_U - T_U)$	work out
III. isothermal compression	$nRT_L \ln \frac{P_3}{P_4} = -nRT_L \ln \frac{P_1}{P_2}$ 3.3&T.3	$-nRT_L \ln \frac{P_3}{P_4} = nRT_L \ln \frac{P_1}{P_2}$ 3.2&T.3	$-nRT_L \ln \frac{P_1}{P_2}$	heat lost at T <sub>L</sub> work in
IV. adiabatic compression	0	$n\bar{C}_V(T_U - T_L)$ 4.4	$-n\bar{C}_V(T_U - T_L)$	work in
net gain/cost	$q_{in} = q_U = q_U$ $+nRT_U \ln \frac{P_1}{P_2}$	$(W_{total})_{sys} = (W_I + W_{II} + W_{III} + W_{IV})_{sys} = -nR(T_U - T_L) \ln \frac{P_1}{P_2}$	$(W_{total})_{surr} = (W_I + W_{II} + W_{III} + W_{IV})_{surr} = - (W_{total})_{sys} = nR(T_U - T_L) \ln \frac{P_1}{P_2}$	$\mathcal{E} = W_{surr}/q_{in}$ $\mathcal{E} = (T_U - T_L)/T_U$

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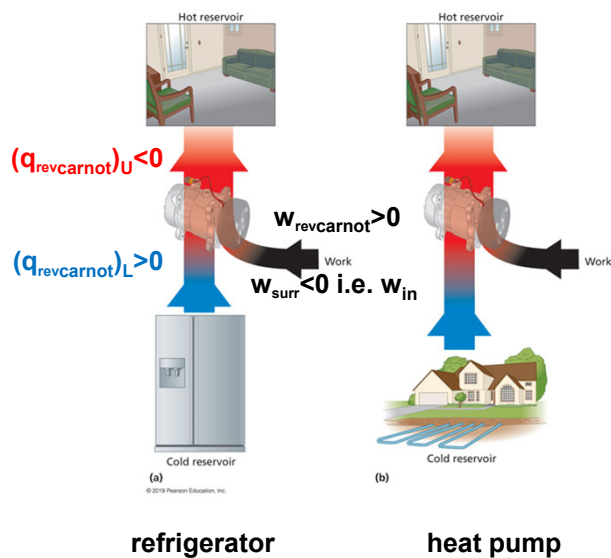
*refrigerators and heat pumps: running the Carnot cycle in REVERSE*

- it only makes 'sense' to talk about running a process in **REVERSE** for **reversible** processes (on a PV diagram there is no 'reverse' process for irreversible expansion against constant  $P_{\text{ext}}$ )
- however the Carnot cycle is a combination of reversible processes

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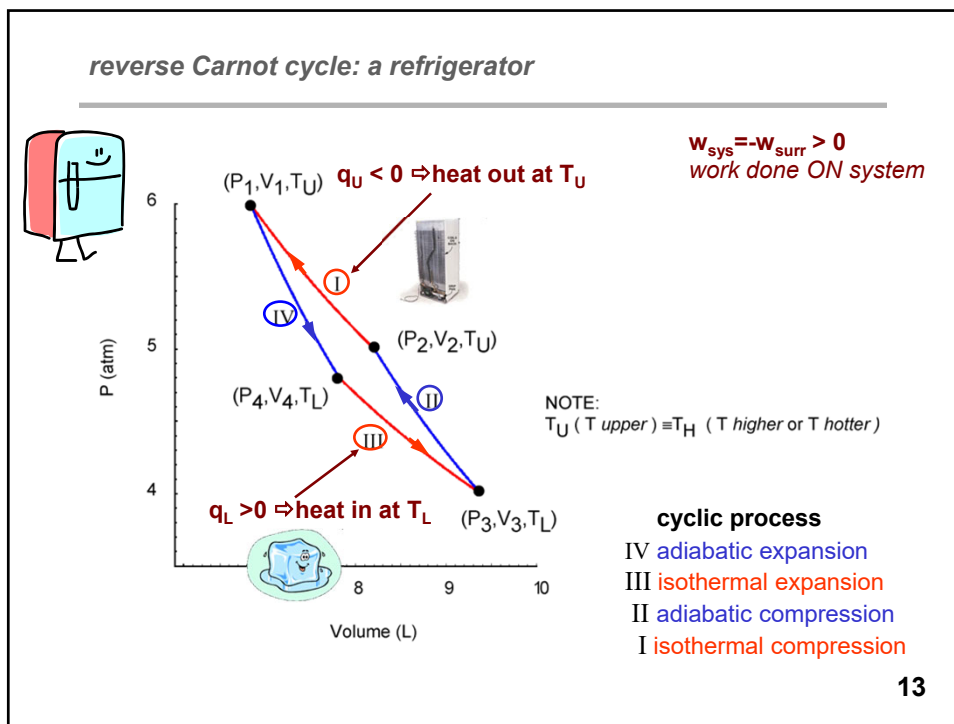
*heat pumps and refrigerators (fig. 5.20 E&R<sub>4th</sub>) 5.17 ER3rd*

Figure 5.20 Application of the principle of a reverse Carnot heat engine to refrigeration and household heating. The reverse Carnot heat engine can be used to induce heat flow from a cold reservoir to a hot reservoir with the input of work. The hot reservoir in both (a) and (b) is a room. The cold reservoir can be configured as the inside of a refrigerator or outside ambient air, earth, or water for a heat pump



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*Carnot refrigerator arithmetic (just negatives of Carnot engine)  
for below table see handout*

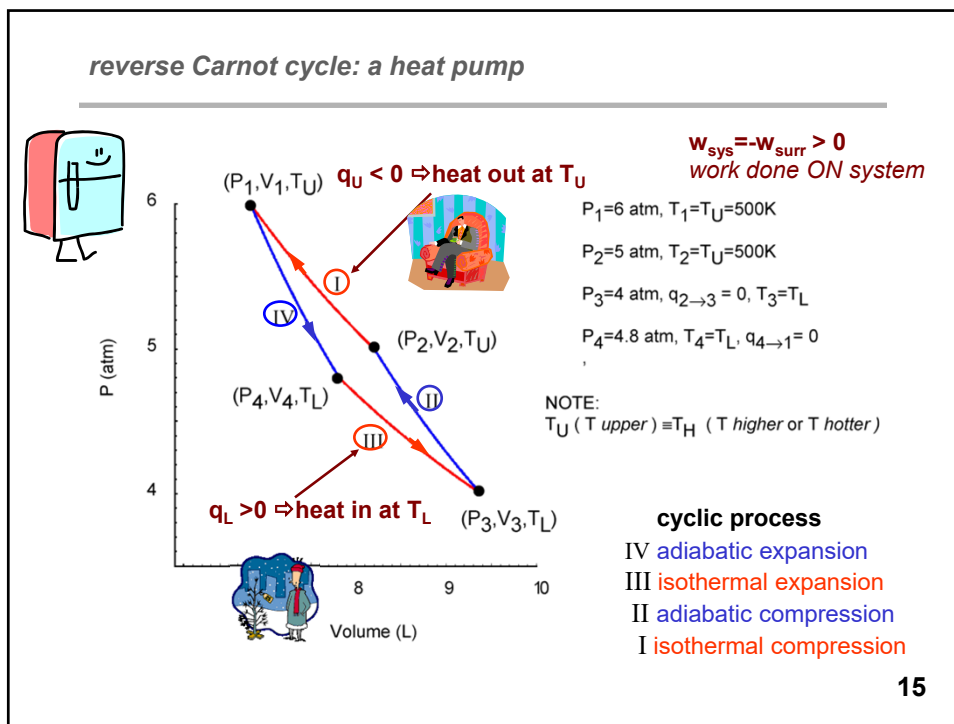
REFRIGERATOR	q	$W_{\text{sys}}$	$q_{\text{surr}}$	
IV. adiabatic expansion $T_H \rightarrow T_L$	0	$-nC_v(T_U - T_L)$	0	work done by system
III. isothermal expansion at $T_L$ $P_4 \rightarrow P_3$	$-nRT_L \ln \frac{P_3}{P_4} = nRT_L \ln \frac{P_1}{P_2}$	$+nRT_L \ln \frac{P_3}{P_4} = -nRT_L \ln \frac{P_1}{P_2}$	$-nRT_L \ln \frac{P_1}{P_2}$	work done by sys heat withdrawn from $T_L$ (ice box)
II. adiabatic compression $T_L \rightarrow T_H$	0	$-nC_v(T_L - T_U)$	0	work on system
I. isothermal compression at $T_H$ $P_2 \rightarrow P_1$	$-nRT_U \ln \frac{P_1}{P_2}$	$+nRT_U \ln \frac{P_1}{P_2}$	$+nRT_U \ln \frac{P_1}{P_2}$	heat out at $T_H$ refrig "coils" work on system
net gain/cost		$W_{\text{total}} = W_I + W_{II} + W_{III} + W_{IV} = +nR(T_U - T_L) \ln \frac{P_1}{P_2}$	$ q_{\text{out}}  = -q_{\text{surr, III}} = +nRT_L \ln \frac{P_1}{P_2}$	$\eta_r \equiv C_R = \frac{-q_{\text{surr, III}}}{W_{\text{sys total}}} = \frac{T_L}{T_U - T_L}$ eqn 5.69 E&R4th [5.45 E&R3rd]

Coefficient of performance of refrigerator:

$$\eta_r \equiv C_R = \frac{-q_{\text{surr, III}}}{W_{\text{total}}} = \frac{q_{\text{III}}}{W_{\text{total}}} = \frac{T_L}{T_U - T_L}$$

eqn 5.69 ER<sub>4th</sub> 5.45 E&R3rd **14**

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*“goodness of performance”*

Since refrigerators and heat pumps are just Carnot machines in reverse, the relationship for  $\epsilon = -w_{\text{total}}/q_I = (T_U - T_L)/T_U$  holds for these cycles. However only for “engines” does it represent the efficiency or “goodness of performance” (work<sub>total out</sub>/heat<sub>in</sub>).

For a refrigerator performance is (heat<sub>in at TL</sub>/work<sub>total</sub>)

$$\eta_r \equiv C_R = \frac{-q_{\text{from freezer}}}{w_{\text{total}}} = \frac{q_{III}}{w_{\text{total}}} = \frac{T_L}{T_U - T_L} \quad \text{eqn 5.69 ER}_{4\text{th}} \quad 5.45 \text{ E\&R3rd}$$

HW5 #28† (E&R<sub>4th</sub> P5.33)

For a heat pump performance is (heat<sub>out at TU</sub>/work<sub>total</sub>)

$$\eta_{hp} \equiv C_{HP} = \frac{q_{\text{given off to room}}}{w_{\text{total}}} = \frac{-q_I}{w_{\text{total}}} = \frac{T_U}{T_U - T_L} \quad (> 1) \quad \text{eqn 5.68 ER}_{4\text{th}} \quad 5.44 \text{ E\&R3rd}$$



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*one more important FACTOID about Carnot machine*

for complete **reversible** Carnot cycle:

$$\oint dU = \Delta U = 0$$

$$\oint dH = \Delta H = 0$$

$$\oint \bar{d}q_{\text{rev}} = q = \neq 0$$

$$\oint \bar{d}w_{\text{rev}} = w = \neq 0$$

BUT ALSO LET'S LOOK AT:

$$\oint_{\text{cycle}} \frac{\bar{d}q_{\text{rev}}}{T}$$

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*one more important FACTOID about Carnot machine*

$$\text{constant } T = T_U \int_I \frac{\bar{d}q_{\text{rev}}}{T} = \frac{nRT_U \ln \frac{P_1}{P_2}}{T_U}$$

$$\text{adiabatic} \int_{II} \frac{\bar{d}q_{\text{rev}}}{T} = 0$$

$$\text{constant } T = T_L \int_{III} \frac{\bar{d}q_{\text{rev}}}{T} = \frac{-nRT_L \ln \frac{P_1}{P_2}}{T_L}$$

$$\text{adiabatic} \int_{IV} \frac{\bar{d}q_{\text{rev}}}{T} = 0$$

$$\oint_{\text{cycle}} \frac{\bar{d}q_{\text{rev}}}{T} = 0$$

have we uncovered a new state function entropy (S) ??

$$\Delta S = \int dS = \int \frac{\bar{d}q_{\text{rev}}}{T}$$

does it just apply  
to ideal gas Carnot cycle ???

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*Generalization of Ideal Gas Carnot Cycle*

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1. Our work on the (**reversible**) Carnot Cycle using an ideal gas as the ‘working substance’ lead to the relationship

$$\varepsilon = \frac{-W_{\text{total}}}{q_U} = \frac{T_U - T_L}{T_U}$$

However, the second law applies to machines with any ‘working substance’ and general cycles.

2. The following presentation shows that a (**reversible**) Carnot machine with ‘any working substance’ **cannot have**  $\varepsilon_{\text{rev any ws}} > \varepsilon_{\text{carnot ideal gas}}$ .

3. **REVERSING** the directions of the ideal gas and ‘any rev Carnot’ (e.g. Raff, pp 160-162) shows that  $\varepsilon_{\text{rev any ws}} = \varepsilon_{\text{carnot ideal gas}}$

4. One can also show (E&R<sup>4th</sup> Fig 5.15 Fig 5.43<sup>3rd</sup>, Raff 162-164) that any Carnot machine has  $\oint dq_{\text{rev}}/T = 0$  and that any reversible machine cycle can be expressed as a sum of Carnot cycles.



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*Carnot machine*

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Carnot machine: **cyclic, reversible, machine** exchanging heat with environment at only two temperatures  $T_U, T_L$

Carnot engine: ‘forward’ Carnot cycle

Carnot heat pump: ‘reverse’ Carnot cycle

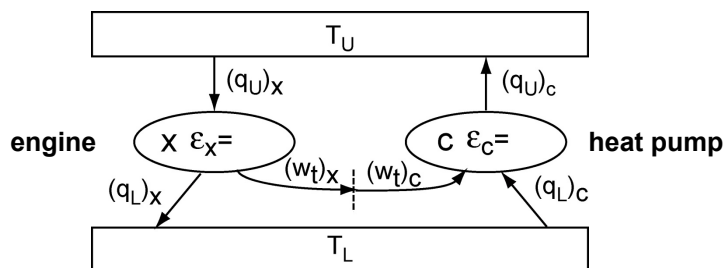
$$\varepsilon = 1 - \frac{T_L}{T_U} = \frac{-w_{\text{total}}}{q_U} \text{ applies to both forward and reverse Carnot cycles}$$

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can  $\epsilon_x > \epsilon_c$  for any Carnot machine X vs ideal gas Carnot machine C ???  
(generalizing  $\epsilon_{rev}=1-T_L/T_U$  that was derived for reversible ideal gas Carnot cycle)

machine X: any 'working substance' (not necessarily ideal gas)  
exchanges heat at only two temperatures  $T_U$  and  $T_L$

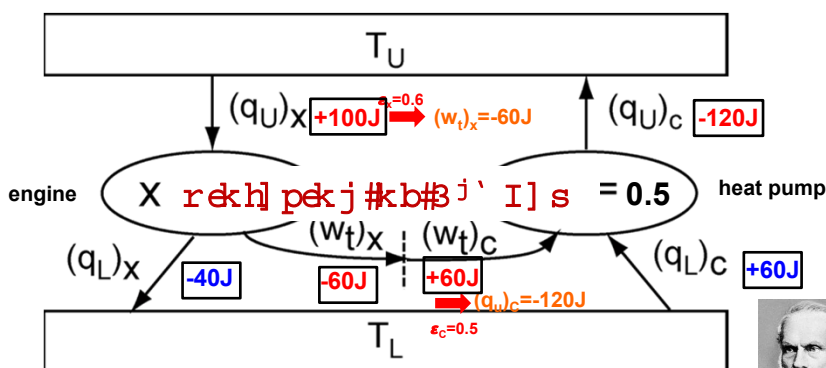


the machine X operates as a Carnot engine doing work, while  
the machine C operates as a reverse Carnot ideal gas engine  
acting as a heat pump.  
In this example the work **done by X**,  $(w_t)_x$  is totally **used by C**:  
 $(w_t)_c = -(w_t)_x$

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CAN  $\epsilon_x > \epsilon_{Carnot}$ : worksheet for **numerical example** (all  $w_x$  goes into  $w_{Carnot}$ )

TRY:  $\epsilon_x = 0.6 > \epsilon_{Carnot} = 0.5$ ; specify heat into x  $(q_U)_x = 100$  J and all  $(w_{out})_x = (w_{in})_{Carnot}$



$$\epsilon = \frac{-W_t}{q_U} \quad q_U = \frac{-W_t}{\epsilon} \quad \Delta U = q_U + q_L + w_t = 0$$

$$q_U \quad w_t = -q_U \epsilon$$

$w_{total} = 0$

$(q_U)_{total} = -20$  J (into  $T_U$ )  
 $(q_L)_{total} = +20$  J (out of  $T_L$ )



Clausius



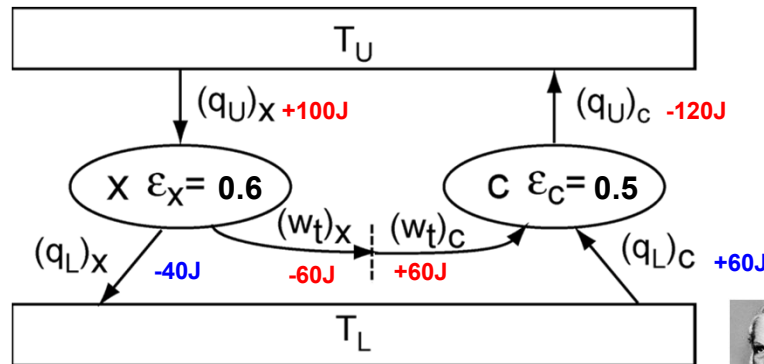
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for each of the cyclic processes

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CAN  $\epsilon_x > \epsilon_{\text{Carnot}}$ : worksheet for **numerical example** (all  $w_x$  goes into  $w_{\text{Carnot}}$ )

TRY:  $\epsilon_x = 0.6 > \epsilon_{\text{Carnot}} = 0.5$ ; specify heat into x  $(q_U)_x = 100 \text{ J}$  and all  $(w_{\text{out}})_x = (w_{\text{in}})_{\text{Carnot}}$



Clausius

$$\epsilon = \frac{-w_t}{q_U} \quad q_U = \frac{-w_t}{\epsilon} \quad \Delta U = q_U + q_L + w_t = 0$$

$$q_U \quad w_t = -q_U \epsilon$$

$w_{\text{total}} = 0$

$(q_U)_{\text{total}} = -20 \text{ J (into } T_U)$   
 $(q_L)_{\text{total}} = +20 \text{ J (out of } T_L)$

for each of the cyclic processes

rek] pckj #kb#B j' I] s

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Clausius Statement of Second Law (numerical example)

we assumed an  $\epsilon_x > \epsilon_{\text{Cig}}$  for ideal gas Carnot and calculated

$w_{\text{total}} = 0$

$(q_L)_{\text{total}} = +20 \text{ J (into 'combo engine' out of cold reservoir at } T_{\text{lower}})$

$(q_U)_{\text{total}} = -20 \text{ J (out of 'combo engine' into heat reservoir at } T_{\text{upper}})$

**BUT:** It is impossible to have a natural process which produces no other effect than absorption of heat from a colder body and discharge of heat to a warmer body. *Clausius's Statement*

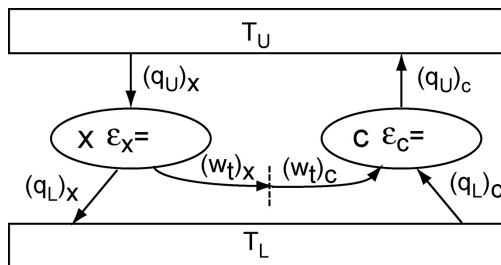
specific example showed:

**if second law holds  $\epsilon_x > \epsilon_{\text{Cig}}$  cannot be true for x = any Carnot ideal gas or not**

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**general proof:** can  $\epsilon_x > \epsilon_c$  for Carnot machine X vs ideal gas Carnot machine C ???



$$\begin{aligned} (q_U)_x &> 0 & (q_U)_c &< 0 \\ \epsilon_x &= \frac{-(w_t)_x}{(q_U)_x} & \epsilon_c &= \frac{-(w_t)_c}{(q_U)_c} = \frac{(w_t)_x}{(q_U)_c} \\ (w_t)_x &= -(q_U)_x \epsilon_x & (w_t)_x &= (q_U)_c \epsilon_c \\ (q_L)_x + (q_U)_x + (w_t)_x &= 0 & (q_L)_c + (q_U)_c + (w_t)_c &= 0 \end{aligned}$$

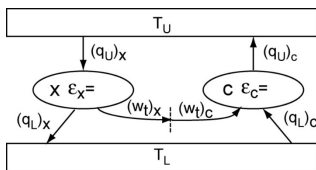
to give  $-(w_t)_x = (w_t)_c$ ,  $\frac{-(q_U)_x}{(q_U)_c} = \frac{\epsilon_c}{\epsilon_x}$ , and  $(q_U)_c = -\frac{\epsilon_x}{\epsilon_c} (q_U)_x$

$$(q_U)_{total} = (q_U)_x + (q_U)_c = (q_U)_x \left(1 - \frac{\epsilon_x}{\epsilon_c}\right) \quad (\text{which will be } < 0, \text{ for } \epsilon_x > \epsilon_c)$$

$$(q_L)_{total} = (q_L)_x + (q_L)_c = -(q_U)_x - (w_t)_x + (-(q_U)_c + (w_t)_x) = -(q_U)_{total}$$

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**general proof:** can  $\epsilon_x > \epsilon_c$  for Carnot machine X vs ideal gas Carnot machine C ???



So  $(q_U)_{total} < 0$

and

$(q_L)_{total} = - (q_U)_{total} > 0$

with  $w_{total} = (w_t)_x + (w_t)_c = 0$

ok # d] p o # p d a # l n k ^ h a i # ==

q transferred  $T_L \rightarrow T_U$  with no net work  
violates Clausius statement of second law

ok # d] p o # p d a # b k t p e k j # ==

$\epsilon_x > \epsilon_c$  is not a condition consistent with the Second Law  
so any Carnot machine, ideal gas or not

$$\epsilon_{\text{any carnot}} \leq \epsilon_{\text{carnot ideal gas}} = \frac{(T_U - T_L)}{T_U}$$

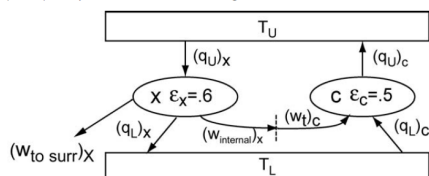
so far

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# Chemistry 163B Winter 2020 Refrigerators and Generalization of Carnot Cycle

## HW#4 Prob. 23

23. The diagram below, where Carnot engine X and Carnot heat pump C are coupled, is similar to that used in lecture to illustrate that the efficiency of *any* Carnot engine,  $\epsilon_x$ , has to be the same as that of an ideal gas Carnot engine,  $\epsilon_c$ , when the engines operate between the same two temperatures. The diagram below differs in that now 33% (-20 J) of the total work done by engine X is done on the surroundings, while 67% (-40 J) is input into the Carnot refrigerator C.



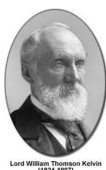
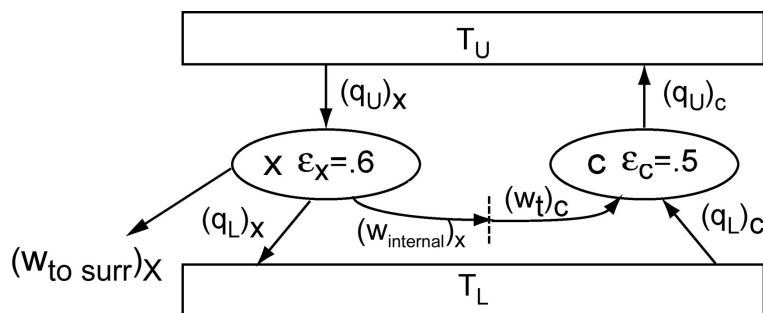
Assume that as indicated  $\epsilon_x = 0.6$  and  $\epsilon_c = 0.5$ .  
For  $(q_U)_x = 100\text{J}$ ,  $(W_{\text{surr}})_x = -20\text{J}$ ,  $(W_i)_x = -40\text{J}$ ,  $(W_t)_c = +40\text{J}$ , and  $(q_U)_c = -80\text{J}$   
**Calculate:**

- $(q_L)_x$
- $(q_L)_c$
- $(q_U)_{\text{total}} = (q_U)_x + (q_U)_c$
- $(q_L)_{\text{total}} = (q_L)_x + (q_L)_c$
- $W_{\text{total}} = (W_{\text{surr}})_x + (W_i)_x + (W_t)_c$
- Considering the [correct] results for parts c, d, and e, how does the process with coupled Carnot engines X and C having  $\epsilon_x = 0.6$  and  $\epsilon_c = 0.5$  violate one of the statements of the Second Law of Thermodynamics

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### setup for (HW prob #23) $\epsilon_x$ efficiency greater than Carnot

TRY:  $\epsilon_x = 0.6 > \epsilon_{\text{Carnot}} = 0.5$ ; specify heat into x  $(q_U)_x = 100\text{J}$  and all  $(2/3)(W_{\text{out}})_x = (W_{\text{in}})_{\text{Carnot}}$



Lord William Thomson Kelvin (1824-1907)

$$(q_U)_x = 100\text{J} \xrightarrow{\epsilon_x = 0.6} (W_{\text{total}})_x = -60\text{J}$$

$$(W_{\text{total}})_c = +40\text{J} \xrightarrow{\epsilon_c = 0.5} (q_U)_c = -80\text{J}$$

$$(W_{\text{sys to surr}})_x = -20\text{J} \quad (W_{\text{internal to C}})_x = -40\text{J}$$

$$\text{calc: } (q_L)_x, (q_L)_c, (q_L)_{\text{total}}, (q_U)_{\text{total}}, (W)_{\text{total}}$$

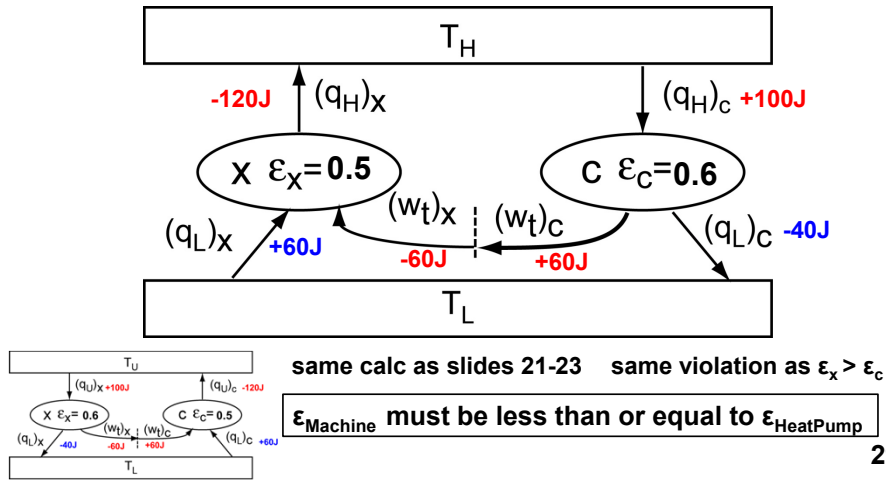
**ANOTHER CONSEQUENCE OF ASSUMING  $\epsilon_x > \epsilon_C$  (2nd Law Statements)**

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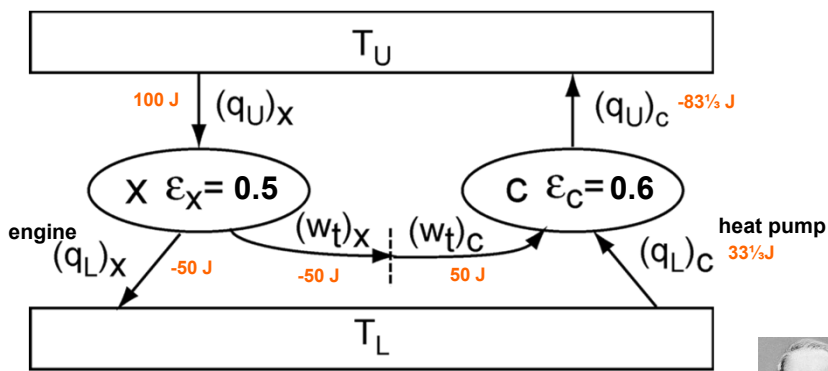
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CAN  $\epsilon_x = 0.5 < \epsilon_c = 0.6$  if X is a some cyclic **REVERSIBLE** machine

**NO:** for reversible X and reversible Carnot,  
**WHY:** can now run X as heat pump  $\epsilon_x = 0.5$  and Carnot as machine  $\epsilon_c = 0.6$



$\epsilon_x > \epsilon_c$  (no way !!; slides #21-#24); **but can  $\epsilon_x < \epsilon_c$  ( $\epsilon_x = 0.5 < \epsilon_c = 0.6$ ) ??**



derivation a la slide #21 with input of  $(q_u)_x = 100\text{J}$   
gives  $(q_u)_{\text{total}} = +16\%$  (out of  $T_U$ )  
 $(q_L)_{\text{total}} = -16\%$  (into  $T_L$ )  $\rightarrow$  heat from hotter to cooler



Clausius

$\epsilon_x = 0.5 < \epsilon_c = 0.6$  is **OK (no violation)**  
if X is machine and C is heat pump

(real engine can be *less* efficient than reversible Carnot heat pump) 30

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*moral of the story for machines exchanging heat at  $T_U$  and  $T_L$*

- $\epsilon_{\text{reversible Carnot ideal gas}} = (1 - T_L/T_U)$
- $\epsilon_{\text{reversible any working substance}}$  **cannot be >**  $\epsilon_{\text{reversible Carnot ideal gas}}$  (slides 21-27)
- $\epsilon_{\text{reversible any working substance}}$  **cannot be <**  $\epsilon_{\text{reversible Carnot ideal gas}}$  (slide 29)
- $\epsilon_{\text{reversible IN GENERAL}} = \epsilon_{\text{reversible Carnot ideal gas}} = (1 - T_L/T_U)$
- $(\epsilon_{\text{irreversible}})_{\text{engine}} < \epsilon_{\text{maximum}} = (1 - T_L/T_U)$  **is OK** (slide 30)

$$\epsilon_{\text{maximum}} = \epsilon_{\text{reversible}} = (1 - T_L/T_U)$$

*general*

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*generalization to arbitrary cycle (E&R Fig. 5.4, Raff Fig. 4.11)*

**WHAT about reversible cycle in general (maybe not only two T's) ?**

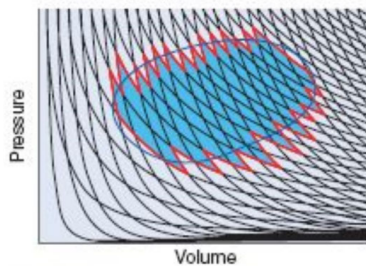
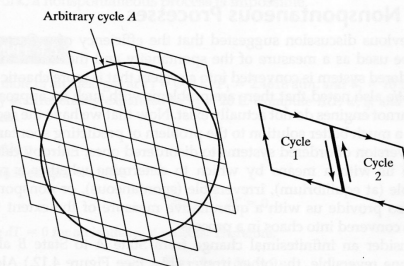


FIGURE 5.4

An arbitrary reversible cycle, indicated by the ellipse, can be approximated to any desired accuracy by a sequence of alternating adiabatic and isothermal segments.



**sum of Carnot cycles**

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a **REALLY BIG RESULT**: connecting  $\varepsilon$  and entropy

---

have shown generally for any reversible **CYCLIC** engine operating between  $T_U$  and  $T_L$ :

$$\varepsilon = \frac{-w_{total}}{q_U} = 1 - \frac{T_L}{T_U} \quad \text{now} \quad -w_{total} = q_U + q_L$$

$$\text{so} \quad \varepsilon = \frac{q_U + q_L}{q_U} = 1 + \frac{q_L}{q_U}$$

thus

$$1 + \frac{q_L}{q_U} = 1 - \frac{T_L}{T_U}$$

$$\frac{q_L}{T_L} + \frac{q_U}{T_U} = 0 \quad \text{FOR THE CYCLE}$$

(ANY REVERSIBLE cyclic process operating between  $T_U$  and  $T_L$ ) **33**

the **BOTTOM LINE**

---

so generally for this reversible cycle

**DEFINE:**  $dS \equiv \frac{\bar{d}q_{reversible}}{T}$

$$\oint dS = \oint \frac{\bar{d}q_{reversible}}{T} = 0$$

and **S** is **STATE FUNCTION**

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# Chemistry 163B Winter 2020

## Refrigerators and Generalization of Carnot Cycle

### roadmap for second law

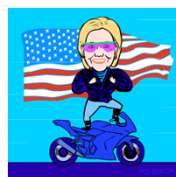
- ✓ 1. Phenomenological statements (what is ALWAYS observed)
- ✓ 2. Ideal gas Carnot [reversible] cycle efficiency of heat  $\rightarrow$  work (Carnot cycle transfers heat only at  $T_U$  and  $T_L$ )
- ✓ 3. Any cyclic engine operating between  $T_U$  and  $T_L$  must have an equal or lower efficiency than Carnot OR VIOLATE one of the phenomenological statements (observations)
- ✓ 4. Generalize Carnot to any reversible cycle (E&R fig 5.4)

- ✓ 5. Show that for this REVERSIBLE cycle

$$q_U + q_L \neq 0 \text{ (}\delta q \text{ inexact differential)}$$

but

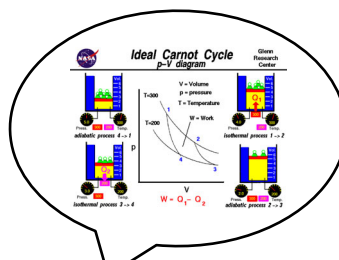
$$\frac{q_U}{T_U} + \frac{q_L}{T_L} = 0 \text{ (something special about } \frac{\delta q_{rev}}{T} \text{)}$$



- ✓ 6. **STATE FUNCTION S**, entropy and spontaneous changes (more to come)

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### did Sadi imagine his ideal gas Carnot Cycle?



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Chemistry 163B Winter 2020  
Refrigerators and Generalization of Carnot Cycle

4th IASME/WSEAS International Conference on ENERGY, ENVIRONMENT, ECOSYSTEMS and SUSTAINABLE DEVELOPMENT (EEESD'08)

Algarve, Portugal, June 11-13, 2008

**Sadi Carnot's Ingenious Reasoning  
of Ideal Heat Engine Reversible Cycles**

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**1. Introduction: Sadi Carnot's Far-  
Reaching Treatise of Heat Engines Was  
Not Noticed at His Time and Even Not  
Fully Recognized Nowadays**

Sadi Carnot laid ingenious foundations for the Second Law of Thermodynamics before the First Law of energy conservation was known and long before Thermodynamic concepts were established. Sadi Carnot may have not been aware of ingenuity of his reasoning, and we may have never known since he died at age 36 from cholera epidemic.

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## 2. Sadi Carnot's Ingenious Reasoning of Ideal Heat Engine Reversible Cycles

54 *MOTIVE POWER OF HEAT.*

The operations which we have just described might have been performed in an inverse direction and order. There is nothing to prevent forming vapor with the caloric of the body *B*, and at the temperature of that body, compressing it in such a way as to make it acquire the temperature of the body *A*, finally condensing it by contact with this latter body, and continuing the compression to complete liquefaction.

The most importantly, Carnot introduced the reversible processes and cycles and, with ingenious reasoning, proved that maximum heat engine efficiency is achieved by any reversible cycle (thus all must have the same efficiency), i.e.:


*"The motive power of heat is independent of the agents employed to realize it; its quantity is fixed solely by the temperatures of the bodies between which is effected, finally, the transfer of the caloric."*  
[1], i.e.:

$$W = W_{netOUT} = Q_{IN} \cdot f_c(T_H, T_L)$$

$$\eta_{Cl} = \frac{W_{netOUT}}{Q_{IN}} \Bigg|_{Max} = \underbrace{f_c(T_H, T_L)}_{Qualitative\ function} \Bigg|_{Rev.} \quad (1)$$

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### Carnot vs Einstein ??? (Milivoje Kostic)



$$\{Q_H, Q_L, W_C\} \overset{\text{IF REVERSED}}{\rightleftharpoons} \{-Q_H, -Q_L, -W_C\}$$

*Carnot*

$$\eta_c = \frac{W}{Q_{IN}} = \underbrace{f_c(T_H, T_L)}_{Qualitative\ function} \Bigg|_{Rev.}$$


$$\frac{Q(T)}{Q(T_0)} = \frac{f(T)}{f(T_0)} \Bigg|_{f(T)=T} = \frac{T}{T_0} = \frac{Q}{Q_0}$$

Carnot Ratio Equality  
(by Carnot's followers)

< ? >

*Einstein*

$$E = mc^2$$



**Fig. 8:** Significance of the Carnot's reasoning of reversible cycles is in many ways comparable with the Einstein's relativity theory in modern times. The *Carnot Ratio Equality* is much more important than what it appears at first. It is probably the most important equation in Thermodynamics and among the most important equations in natural sciences.

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## End of Lecture 10

*(four steps to exactitude)*

