

Lecture 13
Chemistry 163B
Winter 2020

Absolute Entropies and Entropy of Mixing

APPENDIX A: ΔH_f° , ΔG_f° , BUT S° (no Δ , no "sub f")

Substance	ΔH_f° (kJ mol ⁻¹)	ΔG_f° (kJ mol ⁻¹)	S° (J mol ⁻¹ K ⁻¹)	$C_{P,m}^\circ$ (J mol ⁻¹ K ⁻¹)	Atomic or Molecular Weight (amu)
Carbon					
Graphite(s)	0	0	5.74	8.52	12.011
Diamond(s)	1.89	2.90	2.38	6.12	12.011
C(g)	716.7	671.2	158.1	20.8	12.011
CO(g)	-110.5	-137.2	197.7	29.1	28.011
Hydrogen					
H ₂ (g)	0	0	130.7	28.8	2.016
H ₂ O(g)	-241.8	-228.6	188.8	33.6	18.015
H ₂ O(l)	-285.8	-237.1	70.0	75.3	18.015
H ₂ O(s)			48.0	36.2 (273 K)	18.015
H ₂ O ₂ (g)	-136.3	-105.6	232.7	43.1	34.015
H ⁺ (aq)	0	0	0		1.008
OH ⁻ (aq)	-230.0	-157.24	-10.9		17.01
Oxygen					
O ₂ (g)	0	0	205.2	29.4	31.999
O(g)	249.2	231.7	161.1	21.9	15.999
O ₃ (g)	142.7	163.2	238.9	39.2	47.998
OH(g)	39.0	34.22	183.7	29.9	17.01
OH ⁻ (aq)	-230.0	-157.2	-10.9		17.01

Third Law of Thermodynamics

**The entropy of any perfect crystalline substance
approaches 0 as $T \rightarrow 0\text{K}$**

$$S = k \ln W$$

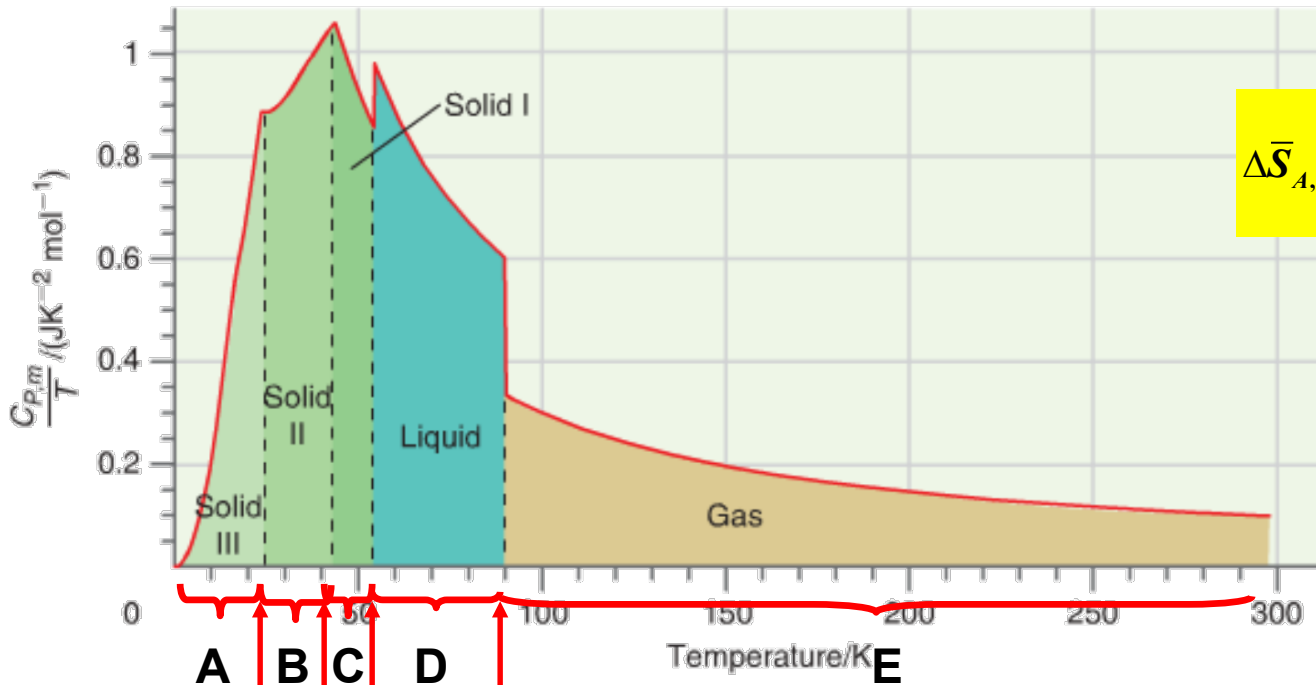


for perfectly ordered crystalline substance

$$W \rightarrow 1 \text{ as } T \rightarrow 0\text{K} \Rightarrow S \rightarrow 0$$



to calculate absolute entropy from measurements
 (E&R_{4th} pp. 119-122 [101-103]_{3rd}, Figs 5.8-5.10)



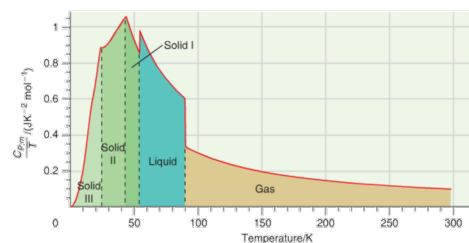
$$\Delta \bar{S}_{A,B,C,D,E} = \int_{T_1}^{T_2} \frac{(\bar{C}_P)_{A,B,C,D,E}}{T} dT$$

$$\Delta \bar{S}_\phi = \frac{\Delta \bar{H}_\phi}{T_\phi}$$

$$\begin{matrix} \Delta \bar{S}_\phi & \Delta \bar{S}_\phi & \Delta \bar{S}_\phi & \Delta \bar{S}_\phi \\ \text{III} \rightarrow \text{II} & \text{II} \rightarrow \text{I} & \text{I} \rightarrow \text{l} & \text{l} \rightarrow \text{g} \end{matrix}$$

$$\begin{aligned} \bar{S}_{298.15}^0 = \bar{S}(0K) &+ \Delta \bar{S}_A &+ \Delta \bar{S}_\phi &+ \Delta \bar{S}_B &+ \Delta \bar{S}_\phi \\ &0 \rightarrow 23.66 &\text{III} \rightarrow \text{II} &23.66 \rightarrow 43.76 &\text{II} \rightarrow \text{I} \\ &&\text{at } 23.66\text{K} &&\text{at } 43.76\text{K} \\ &+ \Delta \bar{S}_C &+ \Delta \bar{S}_\phi &+ \Delta \bar{S}_D &+ \Delta \bar{S}_\phi &+ \Delta \bar{S}_E \\ &43.76 \rightarrow 54.39 &\text{I} \rightarrow \text{l} &54.39 \rightarrow 90.20 &\text{l} \rightarrow \text{g} &90.30 \rightarrow 298.15 \\ &&\text{at } 54.39\text{K} &&\text{at } 90.20\text{K} \end{aligned}$$

full calculation of S°_{298} for $O_2(g)$ (Example Problem 5.7, p123_{4th} [5.9, E&R pp103-104]_{3rd})



	$\Delta \bar{S} \text{ J K}^{-1} \text{ mol}^{-1}$
$\bar{S}(0K)$	0
$\Delta \bar{S}_A (0 \rightarrow 23.66)$	8.182 (1.534+6.649)
$\Delta \bar{S}_\phi (\text{III} \rightarrow \text{II at } 23.66K)$	3.964
$\Delta \bar{S}_B (23.66 \rightarrow 43.76)$	19.61
$\Delta \bar{S}_\phi (\text{II} \rightarrow \text{I at } 43.76K)$	16.98
$\Delta \bar{S}_C (43.76 \rightarrow 54.39)$	10.13
$\Delta \bar{S}_\phi (\text{I} \rightarrow \ell \text{ at } 54.39K)$	8.181
$\Delta \bar{S}_D (54.39 \rightarrow 90.20)$	27.06
$\Delta \bar{S}_\phi (\ell \rightarrow g \text{ at } 90.20K)$	75.59
$\Delta \bar{S}_E (90.20 \rightarrow 298.15)$	35.27
total	204.9 J K⁻¹ mol⁻¹

$\Delta S_{\text{reaction}}$ from absolute entropies



$$\Delta S_{\text{reaction}} = n_C \left(\bar{S}_{298}^0 \right)_C + n_D \left(\bar{S}_{298}^0 \right)_D - n_A \left(\bar{S}_{298}^0 \right)_A - n_B \left(\bar{S}_{298}^0 \right)_B$$

$$\Delta S_{\text{reaction}}^0 (298\text{K}) = \sum_i \nu_i \left(\bar{S}_{298}^0 \right)_i$$

$\left(\bar{S}_{298}^0 \right)_i$ are 3rd Law entropies (e.g. Appendix A)

qualitative factors affecting molecular entropy

- **Higher T** \Rightarrow $\left(\frac{\partial S}{\partial T}\right)_P = \frac{C_P}{T} > 0$
- **Higher P** \Rightarrow $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P < 0$
- **Phase** **S(g) vs S(l) vs S(s)**

(in a reaction the side (reactants vs products) with the greater number of moles of gas generally has higher S

$$\Delta n_{\text{gas}} > 0 \Rightarrow \Delta S_{\text{reaction}} \quad \Delta n_{\text{gas}} < 0 \Rightarrow \Delta S_{\text{reaction}} < 0$$

- **Mixing or dissolving of components**
(l+l), (s+s), (l+s), (g+g) solutions

\Rightarrow

- **(g + l) or (g + s) solution** \Rightarrow

qualitative factors affecting molecular entropy

- **Higher T \Rightarrow Higher S** $\left(\frac{\partial S}{\partial T}\right)_P = \frac{C_P}{T} > 0$
- **Higher P \Rightarrow Lower S** $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P < 0$
- **Phase S(g) > S(l) > S(s)**

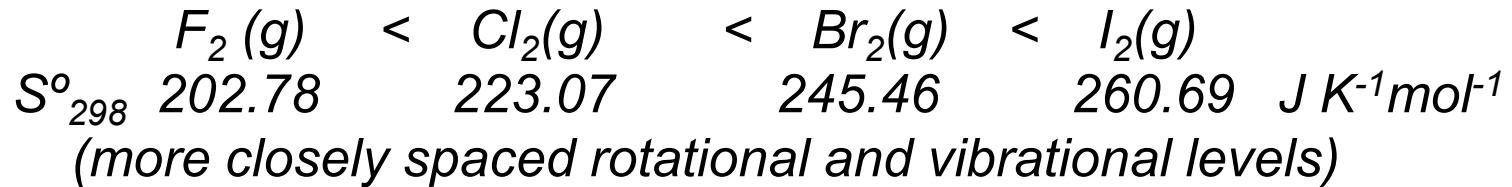
(in a reaction the side (reactants vs products) with the greater number of moles of gas generally has higher S

$$\Delta n_{\text{gas}} > 0 \Rightarrow \Delta S_{\text{reaction}} > 0 ; \Delta n_{\text{gas}} < 0 \Rightarrow \Delta S_{\text{reaction}} < 0$$

- **Mixing or dissolving of components**
(l+l), (s+s), (l+s), (g+g) solutions
 \Rightarrow Higher S
- **(g + l) or (g + s) solution \Rightarrow Lower S**

more qualitative factors affecting molecular entropy

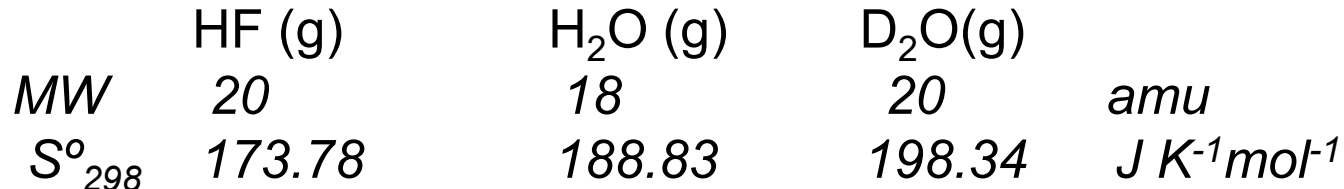
- **substances with higher mass have**



- **more rigid substances have**

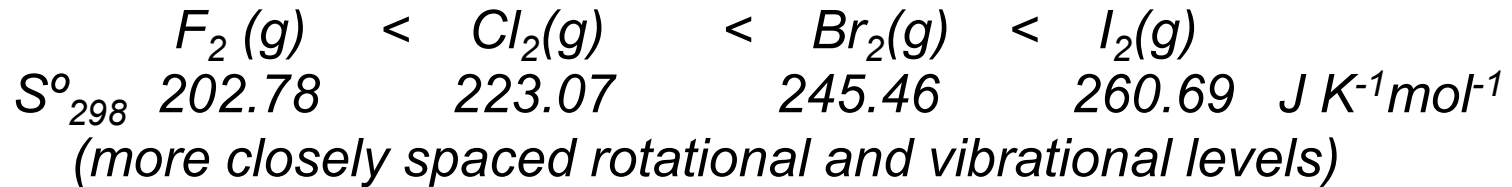


- **more complex substances have**



more qualitative factors affecting molecular entropy

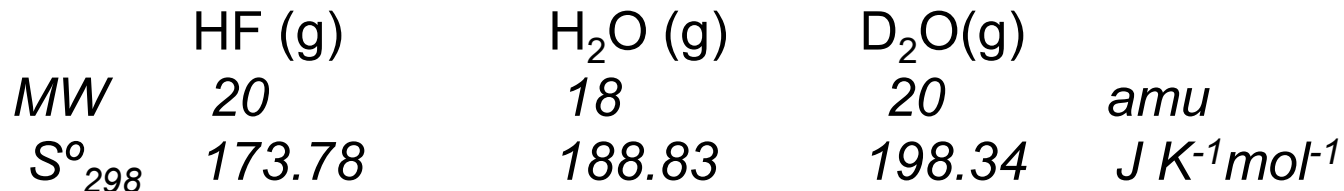
- **substances with higher mass have higher S**



- **more rigid substances have lower S**



- **more complex substances have higher S**



calculating entropy (see summary on review handout)



- Thermal properties of entropy and entropy calculations

✓ ○ $dS = \frac{dq_{rev}}{T}$; $\Delta S = \int \frac{dq_{rev}}{T}$; $\oint \frac{dq_{rev}}{T} = 0$

✓ ○ $\Delta S \geq \int \frac{dq}{T}$; $0 \geq \oint \frac{dq}{T}$; (= for reversible process; > for spontaneous ['real'] process)

✓ ○ $\Delta S_{total \equiv universe} = \Delta S_{system} + \Delta S_{surroundings} \geq 0$

✓ ○ S is a state function; dS is an exact differential

○ Dependence of S on

✓ ■ T: $\left(\frac{\partial \bar{S}}{\partial T}\right)_V = \frac{\bar{C}_V}{T}$; $\left(\frac{\partial \bar{S}}{\partial T}\right)_P = \frac{\bar{C}_P}{T}$

✓ ■ P: $\left(\frac{\partial \bar{S}}{\partial P}\right)_T = -\left(\frac{\partial \bar{V}}{\partial T}\right)_P$

✓ ■ V: $\left(\frac{\partial \bar{S}}{\partial \bar{V}}\right)_T = \left(\frac{\partial P}{\partial T}\right)_{\bar{V}}$

✓ ■ Phase: $\Delta S = \frac{\Delta H_{equilibrium\ phase\ change}}{T_{equilibrium\ phase\ change}}$

○ Calculation of entropy changes for changes in P, V, T, phase

○ Third Law and calculations using Third Law Entropies: $\bar{S}^0(T)$

✓ ○ $\Delta S^0_{reaction}(T) = \sum_i \nu_i \bar{S}_i^0(T)$

○ Entropy of mixing: $\Delta S = -n_{total} R \sum_i X_i \ln X_i$ where $X_i = \frac{n_i}{n_{total}}$

from lecture on 2nd Law and probability (disorder)

(something special about $\frac{dq_{rev}}{T}$)

- Disorder, **W**, did not change during an adiabatic reversible expansion ($q_{rev} = 0$)
- Disorder, **W**, increased in isothermal reversible expansion ($q_{rev} > 0$)
- Disorder, **W**, increased with T increase ($q > 0$)
- Disorder, **W**, decreased with T decrease ($q < 0$)
- As $T \rightarrow 0$, **W** $\rightarrow 1$

Lecture 8, slide 32

3

HW#5 Prob 31

31.

- a. Prove, in general (but $\vec{d}w_{other} = 0$; $dn = 0$), the relationship in equation 3.15 E&R_{4th})

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

Lecture 5 slide 19
“derive later”
now

- b. Evaluate $\left(\frac{\partial U}{\partial V}\right)_T$ for an ideal gas.
- c. Evaluate $\left(\frac{\partial \bar{U}}{\partial \bar{V}}\right)_T$ for a Van der Waals gas $\left(P + \frac{a}{\bar{V}^2}\right)(\bar{V} - b) = RT$
- d. Interpret your (correct!) results for parts (b) and (c).
- e. Prove, in general (but $\vec{d}w_{other} = 0$; $dn = 0$), the relationship in equation 3.40 E&R_{4th}:

$$\left(\frac{\partial H}{\partial P}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_P$$

analogous to 31a

the relationships

definitions:

$U \equiv$ internal energy

$H \equiv U + PV$

$A \equiv U - TS$

$G \equiv H - TS$

differentials of state functions:

$$dU = TdS - PdV$$

$$dH = TdS + VdP$$

$$dA = -SdT - PdV$$

$$dG = -SdT + VdP$$

heat and temperature:

$$\bar{dq}_V = n\bar{C}_V dT \quad \bar{dq}_P = n\bar{C}_P dT$$

$$dU = \bar{dq} + \bar{dw} = \bar{dq} - PdV$$

$$dS = \frac{\bar{dq}_{rev}}{T} \quad dq = Tds$$

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{n\bar{C}_V}{T} \quad \left(\frac{\partial S}{\partial T}\right)_P = \frac{n\bar{C}_P}{T}$$

do some examples:

HW#5 Prob 31a: (and Mid1 2b)
derive E&R_{4th} equation 3.15 'LATER is NOW'

$$\left(\frac{\partial U}{\partial V} \right)_T = ???$$

in terms of P, V, T and their derivatives

technique applies to HW#5 Prob: 31e as well

do another example:

One mole of CO₂(g) is expanded isothermally and reversibly from V₁ to V₂. Using the van der Waals equation of state

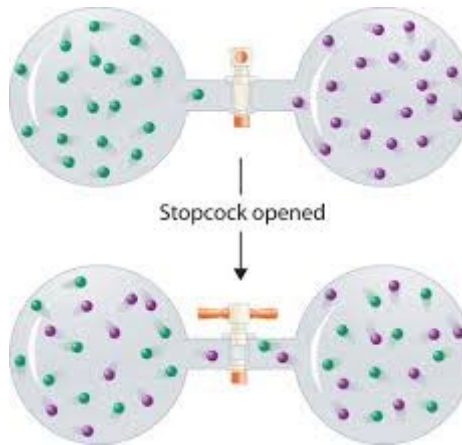
$$\left(P + \frac{a}{\bar{V}^2} \right) (\bar{V} - b) = RT$$

to describe CO₂(g) calculate w, ΔU, q, and ΔS in terms of V₁ and V₂ and the van der Waals constants a and b.

Entropy of Mixing of Distinguishable Ideal Gasses

(EXTRA *but not* OPTIONAL)

$E\&R_{4th} \approx \text{Sec } 6.5 \quad [6.6]_{3rd}$



Entropy of mixing for ideal gas (distinguishable particles)

- isolated from surroundings

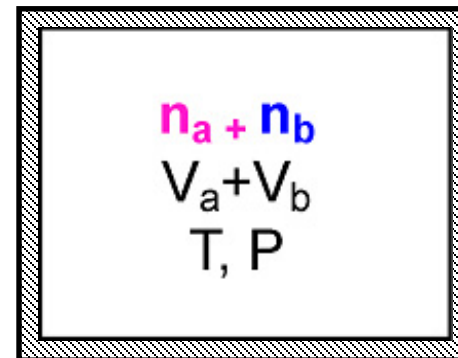
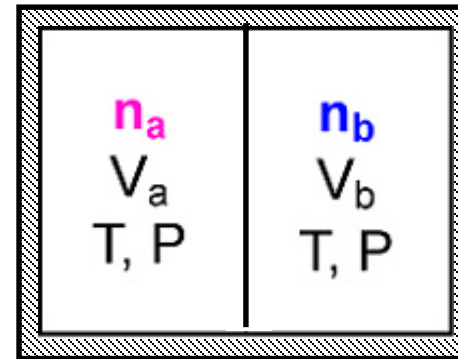
$$q_{\text{sys}} = q_{\text{surr}} = 0$$

$$w = 0$$

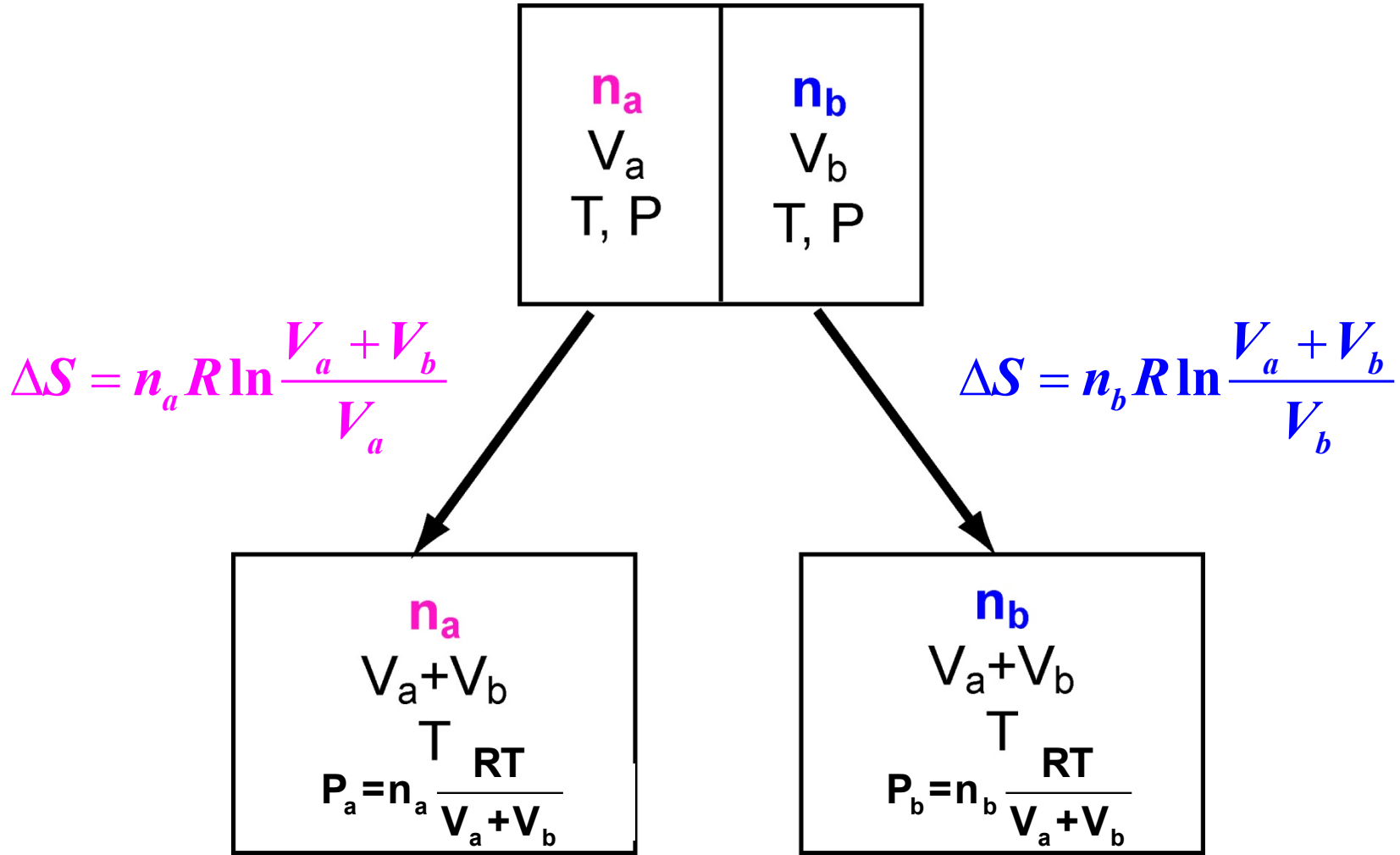
- $\Delta S_{\text{surr}} = 0$

- $\Delta S_{\text{universe}} > 0$

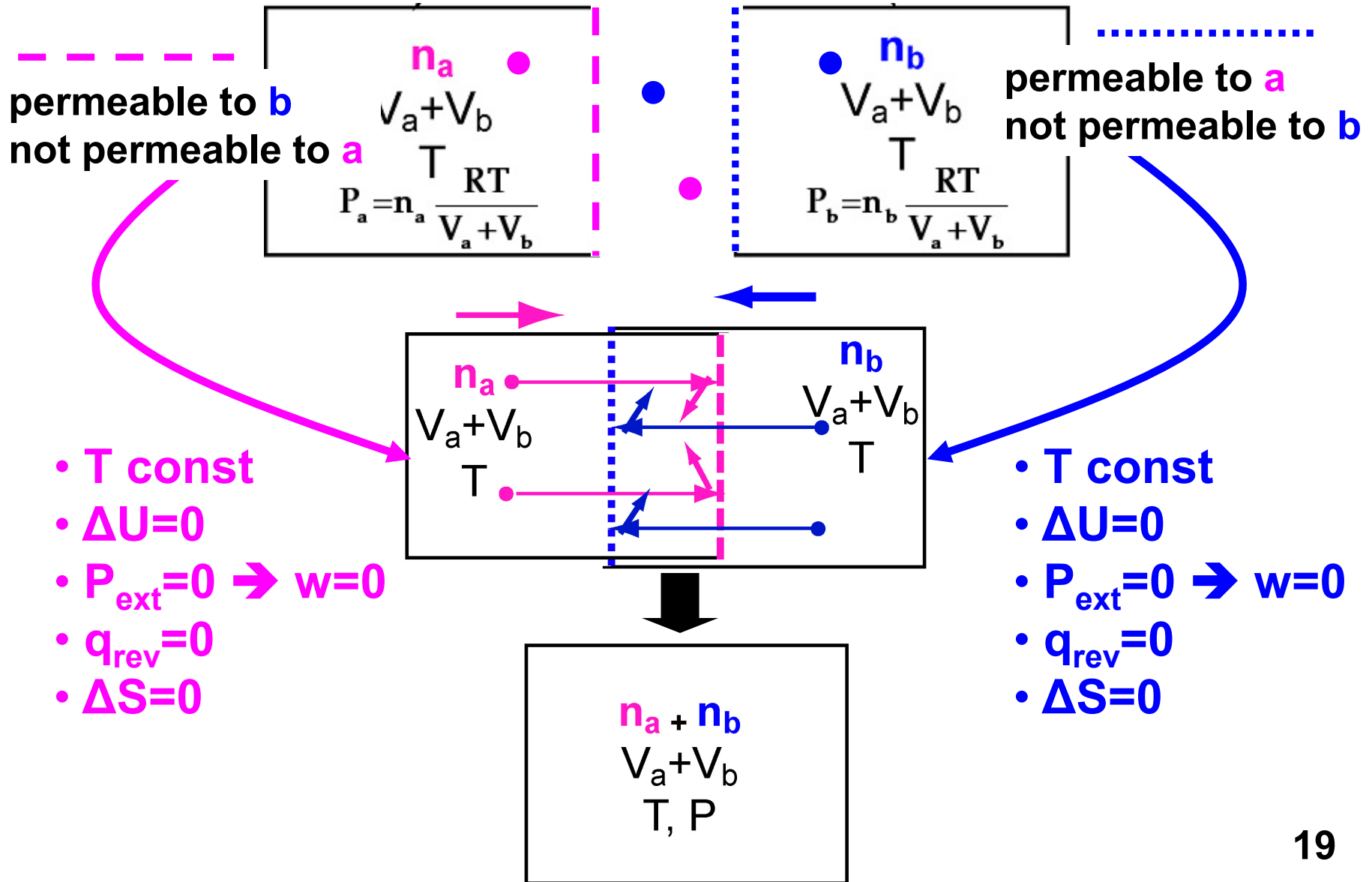
- $\Delta S_{\text{sys}} > 0 = ???$



reversible isothermal expansion to total volume $\Delta S = nR \ln \frac{V_{final}}{V_{initial}}$



reversible recombining of volumes



some arithmetic

note 'flip' leads to $-\ln$

$$\Delta S_{total} = n_a R \ln \frac{V_a + V_b}{V_a} + n_b R \ln \frac{V_a + V_b}{V_b}$$

note 'flip' leads to $-\ln$

(only isothermal expansion steps contribute)

partial pressure : $P_i = \frac{n_i}{n_{total}} P_{total}$ mole fraction : $X_i = \frac{n_i}{n_{total}}$

ideal gas : at start $PV_a = n_a RT$; $PV_b = n_b RT$ ← divide
 at end $P(V_a + V_b) = (n_a + n_b) RT$ ← by

$$\frac{V_a}{V_a + V_b} = \frac{n_a}{n_a + n_b} = X_a \quad \frac{V_b}{V_a + V_b} = \frac{n_b}{n_a + n_b} = X_b$$

$$\Delta S = -n_a R \ln X_a - n_b R \ln X_b$$

and per mole

$$\frac{\Delta S}{n_{total}} = \Delta \bar{S} = -X_a R \ln X_a - X_b R \ln X_b$$

$$\Delta \bar{S}_{mixing} = -R \sum_i X_i \ln X_i$$

$$\Delta S_{mixing} = -n_{total} R \sum_i X_i \ln X_i$$

calculating entropy (see summary on review handout)



- Thermal properties of entropy and entropy calculations
 - ✓ ○ $dS = \frac{\bar{d}q_{rev}}{T}$; $\Delta S = \int \frac{\bar{d}q_{rev}}{T}$; $\oint \frac{\bar{d}q_{rev}}{T} = 0$
 - ✓ ○ $\Delta S \geq \int \frac{\bar{d}q}{T}$; $0 \geq \oint \frac{\bar{d}q}{T}$; (= for reversible process; > for spontaneous ['real'] process)
 - ✓ ○ $\Delta S_{total \equiv universe} = \Delta S_{system} + \Delta S_{surroundings} \geq 0$
 - ✓ ○ S is a state function; dS is an exact differential
 - Dependence of S on
 - ✓ ■ T: $\left(\frac{\partial \bar{S}}{\partial T}\right)_V = \frac{\bar{C}_V}{T}$; $\left(\frac{\partial \bar{S}}{\partial T}\right)_P = \frac{\bar{C}_P}{T}$
 - ✓ ■ P: $\left(\frac{\partial \bar{S}}{\partial P}\right)_T = -\left(\frac{\partial \bar{V}}{\partial T}\right)_P$
 - ✓ ■ V: $\left(\frac{\partial \bar{S}}{\partial \bar{V}}\right)_T = \left(\frac{\partial P}{\partial T}\right)_{\bar{V}}$
 - ✓ ■ Phase: $\Delta S = \frac{\Delta H_{equilibrium\ phase\ change}}{T_{equilibrium\ phase\ change}}$
 - ✓ ○ Calculation of entropy changes for changes in P, V, T, phase
 - Third Law and calculations using Third Law Entropies: $\bar{S}^{\circ}(T)$
 - ✓ ○ $\Delta S_{reaction}^{\circ}(T) = \sum_i \nu_i \bar{S}_i^{\circ}(T)$
 - ✓ ○ Entropy of mixing: $\Delta S = -n_{total} R \sum_i X_i \ln X_i$ where $X_i = \frac{n_i}{n_{total}}$

What formulas I have to memorize for midterm and final ?

Relationships FOR MIDTERM #2 CHEMISTRY 163B

definitions for U, H, A, and G (student fills in at exam time):

total differentials for: dU, dH, dA, and dG (student fills in at exam time):

For reversible adiabatic path, ideal gas:

- $T_1^{\gamma} V_1^{1-\gamma} = T_2^{\gamma} V_2^{1-\gamma}$ (adiabatic reversible path)
- $p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$ (adiabatic reversible path, $PV^{\gamma} = \text{constant}$)
- $\frac{T_2}{T_1} = \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}}$ (adiabatic reversible path)

Energy and enthalpy:

- $(\Delta H_{\text{reaction}})_T = \Delta U_{\text{reaction}} + \Delta n_{\text{gas}} RT$
- $\left(\frac{\partial H}{\partial T}\right)_p = C_p = n\bar{C}_p$; $\Delta H(T_2) = \Delta H(T_1) + \int_{T_1}^{T_2} \Delta C_p dT$

For ideal gas:

- $\Delta U = C_v \Delta T = n\bar{C}_v \Delta T$
- $\Delta H = C_p \Delta T = n\bar{C}_p \Delta T$
- $(\bar{C}_p - \bar{C}_v) = R$
- $\bar{C}_v = \frac{3}{2}R$ (monatomic ideal gas)

For ideal, reversible, Carnot Engine:

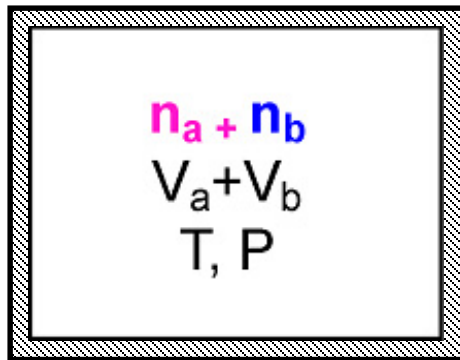
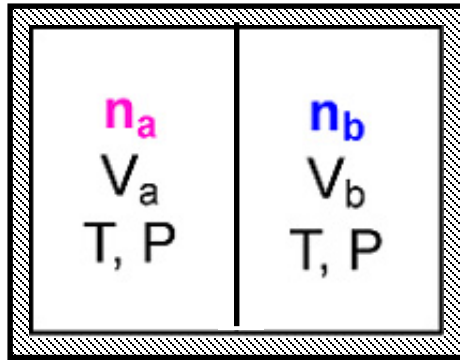
- $e = \frac{-w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{T_c}{T_h}$

Some entropy relationships:

- $\left(\frac{\partial S}{\partial T}\right)_p = \frac{C_p}{T} = n\frac{\bar{C}_p}{T}$; $\left(\frac{\partial S}{\partial T}\right)_v = \frac{C_v}{T} = n\frac{\bar{C}_v}{T}$
- $\Delta S_{\text{mixing, ideal gases}} = -n_{\text{total}} R \sum_i X_i \ln X_i$ (where X_i is mole fraction component i)

[see reverse side for more relationships]:

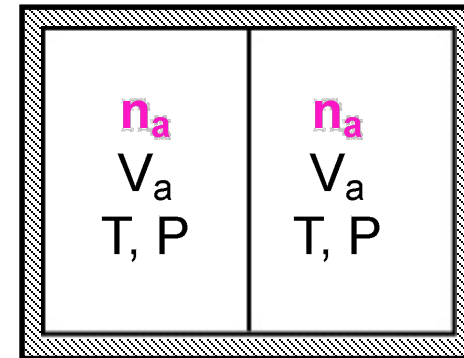
Gibb's Paradox: distinguishable vs identical (don't fret)



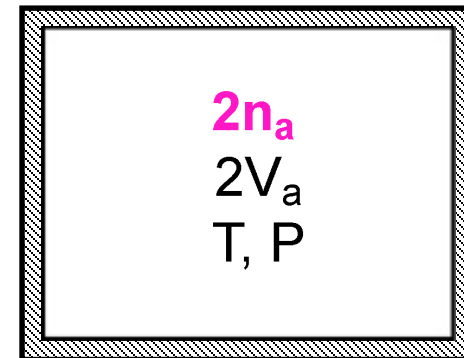
$$n_a = n_b$$

$$V_a = V_b$$

$$\Delta S_{total} = 2n_a R \ln 2$$



identical to
 n_a
 n_b
 $V_a = V_b$



$$\Delta S_{total} = 0$$

$$\Delta S_{total} = n_a R \ln \frac{V_a + V_b}{V_a} + n_b R \ln \frac{V_a + V_b}{V_b}$$

CORRECT

CORRECT TOO

Gibb's Paradox: distinguishable vs identical (don't fret)

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Full Paper

Gibbs' Paradox and the Definition of Entropy

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The Gibbs Paradox and the Distinguishability of Identical Particles

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(Dated: 3 December 2010)

statistics, Chemistry 163C

Some references:

www.mdpi.org/lin/entropy/Chua-Thermodynamics.ppt

<http://bayes.wustl.edu/etj/articles/gibbs.paradox.pdf>

http://www.nyu.edu/classes/tuckerman/stat.mech/lectures/lecture_6/node5.html

<http://www.jstor.org/stable/186086>



where we have been

and

what's next



observations: *thermo* \equiv *heat*

- Count Rumford, 1799
- observed water turning into steam when canon barrel was bored
- *work* \Leftrightarrow *heat*

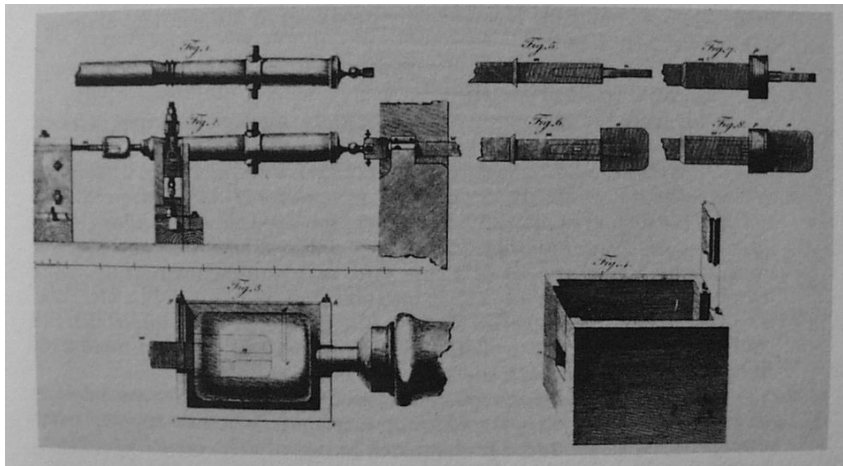
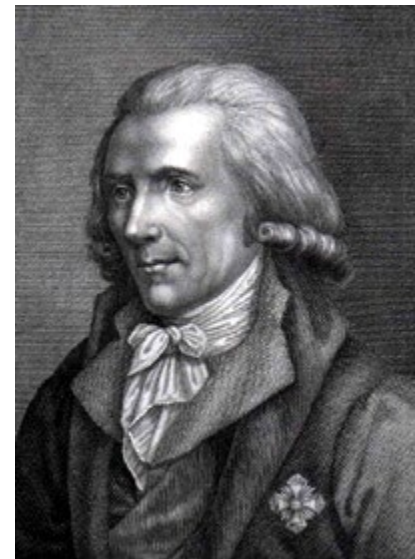
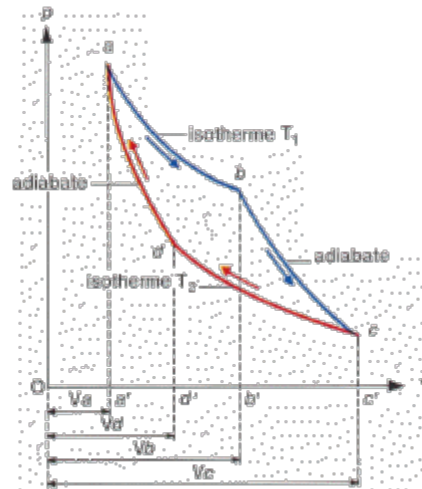
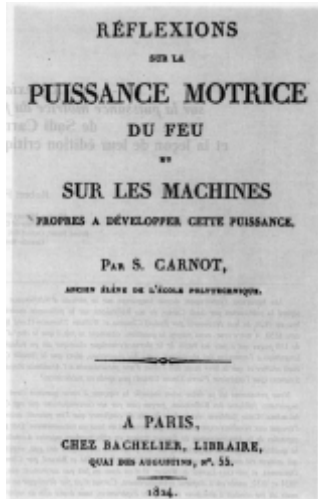


FIGURE 11. An illustration from Rumford's paper, "An Inquiry Concerning the Source of the Heat Which is Excited by Friction," showing the apparatus used by him in the cannon boring experiment. Figure 1, upper left, shows the cannon as received from the foundry, and Figure 2, below, shows it mounted in the machine used for boring. (Reproduced with the permission of Harvard University Press.)



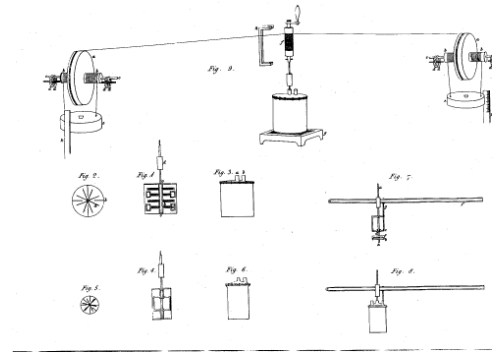
observations: mechanical efficiency of steam engine

- Sadi Carnot, 1824
- efficiency of engines



guiding principles

- Conservation of heat and work (Joule, 1845)
1st LAW OF THERMODYNAMICS



Joule's original water-friction experiment for determining the mechanical equivalent of heat. (Crown copyrights, Science Museum, London.)

- Clausius, 1860 Entropy
2nd LAW OF THERMODYNAMICS



Clausius

- Boltzmann, late 19th century,
molecular picture of entropy



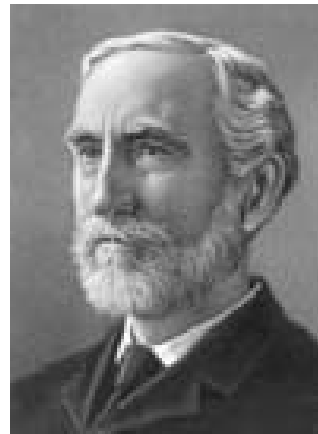
Boltzmann

the thermodynamic functions U, H, and S
(1st and 2nd laws)

“Applications”

How does knowledge about efficiencies of steam engines, mechanical systems, etc, relate to processes in chemical, biological, and geological systems?

ANSWERED BY:



J. W. Gibbs- arguably the first great American scientist who combined the concepts of heat and entropy and proposed “[Gibbs] Free Energy”, G , a thermodynamic state function that leads to a whole spectrum of applications

End of Lecture

In 1872, long before women were admitted to Austrian universities, he met Henriette von Aigentler, an aspiring teacher of mathematics and physics in Graz. She was refused permission to audit lectures unofficially. Boltzmann advised her to appeal, which she did, successfully.

In Vienna, Boltzmann not only taught physics but also lectured on philosophy. Boltzmann's lectures on natural philosophy were very popular, and received a considerable attention at that time. His first lecture was an enormous success. Even though the largest lecture hall had been chosen for it, the people stood all the way down the staircase. Because of the great successes of Boltzmann's philosophical lectures, the Emperor invited him for a reception at the Palace.

On September 5, 1906, while on a summer vacation in Duino, near Trieste, Boltzmann hung himself during an attack of depression.^{[8][9]} He is buried in the Viennese Zentralfriedhof; his tombstone bears the inscription

$$S = k \cdot \log W.$$

To quote Planck, "The logarithmic connection between entropy and probability was first stated by L. Boltzmann in his kinetic theory of gases".^[10] This famous formula for entropy S is^{[11][12]}

$$S = k \log_e W$$

where $k = 1.3806505(24) \times 10^{-23} \text{ J K}^{-1}$ is Boltzmann's constant, and the logarithm is taken to the natural base e . W is the *Wahrscheinlichkeit*, the frequency of occurrence of a macrostate^[13] or, more precisely, the number of possible microstates corresponding to the macroscopic state of a system — number of



Ludwig Boltzmann (standing, from left) and Henriette von Aigentler (sitting, from left). Boltzmann, Klein



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