

Lecture 22-23 Chemistry 163B W2020 **Colligative Properties Challenged Penpersonship Notes**



colligative properties of solutions



colligative One entry found.

Main Entry: col·li·ga·tive Pronunciation: ˈkä-lə-ˌgā-tiv, kə-ˈli-gə-tiv Function: adjective

: depending on the number of particles (as molecules) and not on the nature of the particles ressure is a colligative property>

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quantitative treatment of colligative properties

- A. Freezing point depression
- B. Boiling Point Elevation
- C. Osmotic Pressure

quantitative treatment of colligative properties

Handout #53 Colligative Properties

from relationships for Chem 163B final:

Colligative properties:

• freezing point lowering:

(hopefully) reassuring ??

quantitative treatment of colligative properties

- I. The pure solvent (component B) is originally in equilibrium in the two phases.
- II. Addition of solute (component A) lowers the chemical potential of the solvent in the solution phase
- III. Temperature (freezing point depression, boiling point elevation) or pressure (osmotic pressure) must be altered to reestablish equilibrium between the solution and the pure solvent phase.
- IV. Obtain relationships between X_A or X_B and change in T or P.

It's as easy as I., II., III., IV.

freezing point depression (solid ≠liquid [solution])

I. pure solvent is originally in equilibrium in the two phases

solid is pure "solvent" B

 $X_B = mole \ fraction \ solvent \ B \ in \ solution$

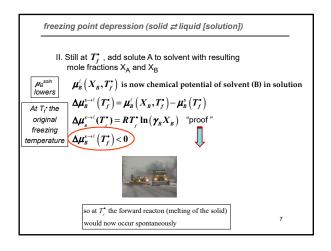
 $X_A = mole fraction solute A in liquid$

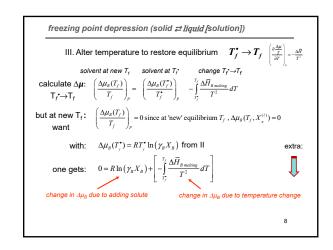
pure $solid_{B}^{\bullet} \rightleftharpoons pure \ \ell iquid_{B}^{\bullet}$ at $T_{\ell}^{\bullet} \equiv$ the normal melting point $\left(T_{fusion}^{\bullet}\right)$

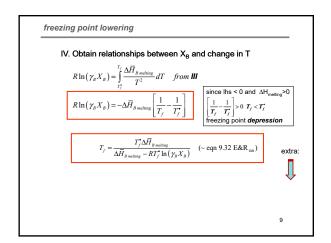
$$\mu_B^{s\bullet}(T_f^{\bullet}) = \mu_B^{\ell\bullet}(X_B = 1, T_f^{\bullet})$$

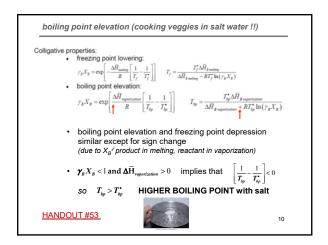
$$\Delta \mu_{\scriptscriptstyle B}(T_{\scriptscriptstyle f}^{\bullet}) = \mu_{\scriptscriptstyle B}^{\ell \bullet}(T_{\scriptscriptstyle f}^{\bullet}) - \mu_{\scriptscriptstyle B}^{s \bullet}(T_{\scriptscriptstyle f}^{\bullet}) = 0$$

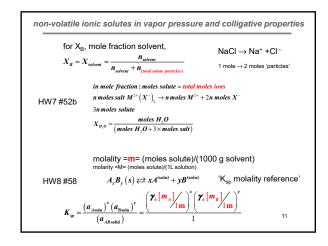
$$\Delta \overline{H}(T_{_{f}}^{\bullet}) = \Delta \overline{H}_{\mathit{B melting}} \quad > 0 \qquad \qquad \textit{for solid} \ \, \rightleftarrows \textit{liquid}$$

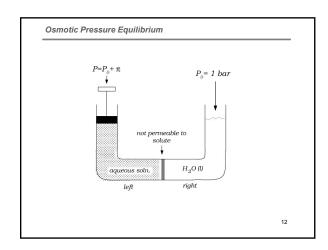












osmotic pressure (solution [solvent + solute] ≠ pure solvent)

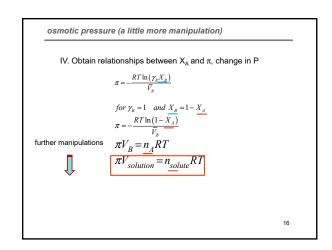
 $\label{eq:local_local_local} I. \quad \text{pure solvent at P_{left} is originally in equilibrium with } \\ \text{pure solvent at P_{right} ; i.e. P_{left}=P_{right}=P_0 }$

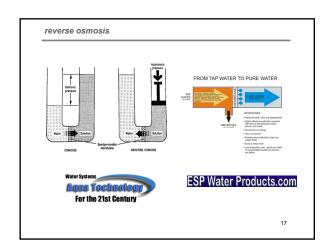
pure $liquid_*(P_0, left) \rightleftharpoons pure \ liquid_*(P_0, right)$ at T left and 'right' refer to compartments separated by solute impermeable membrane $\mu_n^*(P_0, left) = \mu_n^*(P_0, right)$

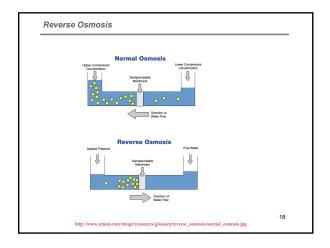
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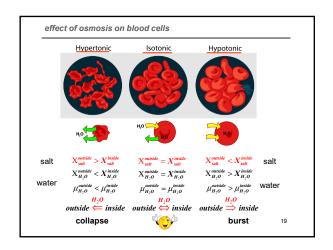
II. in left hand compartment add solute A to solvent with resulting mole fractions X_A and X_B add X_A solute to liquid in 'left' compatment resulting in X_B for solvent lectures 20-21 $\mu_B'(P_0, left) = \mu_B'(P_0, left) + RT \ln(\gamma_B X_B) < \mu_B'(P_0, right)$ so the solvent B moves spontaneously left \leftarrow right (i.e. diluting solution on left)

osmotic pressure (III, alter pressure) III. alter Pressure: $P_{left} \rightarrow (P_0 + \pi)_{left}$ to restore equilibrium $solution (X_B, P_0 + \pi, left) \rightleftharpoons pure solvent(P_0, right)$ want: $\mu_{\text{solution}}(X_B, P_0 + \pi, left) = \mu_{\text{pure solvent}}^{\bullet}(P_0, right)$ $\left(\frac{\partial \mu_{B}^{left}}{\partial P_{left}}\right)_{T} = \overline{V}_{B} \qquad \int\limits_{P_{0}}^{P_{0}+\pi} d\,\mu_{B}^{left}\left(X_{B}\right) = \int\limits_{P_{0}}^{P_{0}+\pi} \overline{V}_{B} dP$ how does μ change with pressure? $\mu_{\scriptscriptstyle B}^{\rm left}\left(X_{\scriptscriptstyle B},P_{\scriptscriptstyle 0}+\pi\right) = \mu_{\scriptscriptstyle B}^{\rm left}\left(X_{\scriptscriptstyle B},P_{\scriptscriptstyle 0}\right) + \left[P_{\scriptscriptstyle 0}+\pi-P_{\scriptscriptstyle 0}\right]\overline{V}_{\scriptscriptstyle B}$ integrate $\mu_{\scriptscriptstyle B}^{\scriptscriptstyle left}\left(X_{\scriptscriptstyle B},P_{\scriptscriptstyle 0}+\pi\right)=\mu_{\scriptscriptstyle B}^{\scriptscriptstyle left}\left(X_{\scriptscriptstyle B},P_{\scriptscriptstyle 0}\right)+\pi\overline{V}_{\scriptscriptstyle B}$ from II. $\mu_B^{left}\left(X_B, P_0 + \pi\right) = \overline{\mu_B^{\bullet}\left(P_0\right)} + RT \ln\left(\gamma_B X_B\right) + \pi \overline{V}_B$ extra: $\underbrace{\mu_{\mathcal{B}}^{\bullet}(P_{0}) + RT \ln(\gamma_{\mathcal{B}}X_{\mathcal{B}}) + \pi \overline{V_{\mathcal{B}}}}_{left} = \underbrace{\mu_{\mathcal{B}}^{\bullet}(P_{0})}_{vight} \quad \boxed{\begin{array}{c} \textit{right with} \\ \textit{P=P}_{0} \end{array}}$ left with $P=P_0+\pi$ right $RT \ln (\gamma_{\scriptscriptstyle B} X_{\scriptscriptstyle B})$ 15











 $\begin{aligned} & \textbf{Handout \#53 } & \textbf{Colligative Properties} \\ & \textbf{From relationships for Chem 163B final:} \\ & \textbf{Colligative properties:} \\ & \textbf{ • freezing point lowering:} \\ & \textbf{ • freezing point lowering:} \\ & \textbf{ • ps}_{X_g} = \exp\left[-\frac{\Delta \overline{H}_{above}}{R}\left[\frac{1}{I_f} - \frac{1}{I_f}\right]\right] & T_f = \frac{T_f^* \Delta \overline{H}_{g-aning}}{\Delta \overline{H}_{g-aning}} - RT_f \ln(\tau_g X_g) \\ & \textbf{ • boiling point elevation:} \\ & \textbf{ • ps}_{X_g} = \exp\left[\frac{\Delta \overline{H}_{aboverinton}}{R}\left[\frac{1}{I_{tp}} - \frac{1}{I_{tp}}\right]\right] & T_{tp} = \frac{T_{tp}^* \Delta \overline{H}_{g-appertation}}{\Delta \overline{H}_{g-appertation}} + RT_{tp}^* \ln(\tau_g X_g) \\ & \textbf{ • osmotic pressure:} \\ & \pi = \frac{-RT \ln(\tau_g X_g)}{\overline{I_g^*}} & \pi \approx \frac{n_g RT}{V_g} = \frac{n_{pobase}RT}{V_{sobover}} & for dilute solution \\ & \textbf{ (hopefully) reassuring ??} \end{aligned}$

End of Lectures 22.23

II. Still at T_f^* , add solute A to solvent with resulting mole fractions X_A and X_B for solid phase of B there is no change: $\mu_B^{**}(T_f^*) = \mu_B^{**old}(T_f^*)$ for the solvent (B) in solution: $S^{**} = I^{**} = I^$

III. Alter temperature to restore equilibrium $T_f^* o T_f$ $\left(\frac{\partial \frac{\Delta \mu}{T}}{\partial T} \right)_p = -\frac{\Delta \overline{H}}{T^2}$ $\int_{\tau_f}^{\tau_f} d\left(\frac{\Delta \mu_B}{T} \right)_p = -\int_{\tau_f}^{\tau_f} \frac{\Delta \overline{H}_{B \, \text{melting}}}{T^2} \, dT \quad \text{old stuff}$ $\left(\frac{\Delta \mu_B(T_f)}{T_f} \right)_p - \left(\frac{\Delta \mu_B(T_f)}{T_f^*} \right)_p = -\int_{\tau_f}^{\tau_f} \frac{\Delta \overline{H}_{B \, \text{melting}}}{T^2} \, dT$

freezing point depression (solid 2 solution)

III. Alter temperature to restore equilibrium (continued)

$$\begin{split} \left(\frac{\Delta\mu_{B}^{s}(T_{f})}{T_{f}}\right)_{p} - \left(\frac{\Delta\mu_{B}^{s}(T_{f}^{*})}{T_{f}^{*}}\right)_{p} &= -\int\limits_{\tau_{f}}^{\tau_{f}} \frac{\Delta\overline{H}_{B \, meling}}{T^{2}} dT \\ \left(\frac{\Delta\mu_{B}(T_{f})}{T_{f}}\right)_{p} &= 0 \text{ since at 'new' equilibrium } T_{f} \text{ , } \Delta\mu_{B}(T_{f}) = 0 \\ and \quad \left(\frac{\Delta\mu_{B}(T_{f}^{*})}{T_{f}^{*}}\right)_{p} &= R \ln\left(\gamma_{B}X_{B}\right) \quad \text{from eqn in II.} \\ -R \ln\left(\gamma_{B}X_{B}\right) &= -\int\limits_{\tau_{f}}^{\tau_{f}} \frac{\Delta\overline{H}_{B \, meling}}{T^{2}} dT \end{split}$$

freezing point lowering

IV. Obtain relationships between X_B and change in T

$$R \ln (\gamma_{\scriptscriptstyle B} X_{\scriptscriptstyle B}) = \int_{T_j}^{T_j} \frac{\Delta \overline{H}_{\scriptscriptstyle B \, melting}}{T^2} dT$$

$$\Delta \overline{H}_{B \text{ melting}} \sim \text{independent of T}$$

$$R\ln\left(\gamma_{\scriptscriptstyle B}X_{\scriptscriptstyle B}\right) = -\Delta \bar{H}_{\scriptscriptstyle B\ melting} \left[\frac{1}{T_{\scriptscriptstyle f}} - \frac{1}{T_{\scriptscriptstyle f}^{\bullet}}\right]$$

since
$$lhs < 0 \Rightarrow T_f < T_f^*$$
 (freezing point **depression**)
$$\gamma_B X_B = \exp\left[-\frac{\Delta \overline{H}_{B \text{ melting}}}{R} \left[\frac{1}{T_f} - \frac{1}{T_f^*}\right]\right] \text{ (integration of eqn 9.31 E&R)}$$

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freezing point lowering

IV. Obtain relationships between X_{B} and change in T (cont)

$$\gamma_{\scriptscriptstyle B} X_{\scriptscriptstyle B} = \exp \left[-\frac{\Delta \overline{H}_{\scriptscriptstyle B \, melting}}{R} \left[\frac{1}{T_{\scriptscriptstyle f}} - \frac{1}{T_{\scriptscriptstyle f}^{\bullet}} \right] \right]$$

$$\begin{split} &-\frac{R}{\Delta \overline{H}_{B \; melting}} \ln \left(\gamma_B X_B\right) + \frac{1}{T_f} = \frac{1}{T_f} \\ &T_f = \frac{T_f \Delta \overline{H}_{B \; melting}}{\Delta \overline{H}_{B \; melting} - RT_f^* \ln \left(\gamma_B X_B\right)} \quad (\sim \text{eqn } 9.32 \; \text{E\&R}_{4th}) \end{split}$$



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osmotic pressure (III, alter pressure)

III. alter Pressure: $P_{left} \rightarrow (P_0 + \pi)_{left}$ to restore equilibrium $solution(X_B, P_0 + \pi, left) \rightleftharpoons pure solvent(P_0, right)$

$$\left(\frac{\partial \mu_B^{left}}{\partial P_{left}}\right) = \overline{V}_B$$

$$\int\limits_{P_{0}}^{P_{0}+\pi}d\mu_{B}^{left}\left(X_{B}\right)=\int\limits_{P_{0}}^{P_{0}+\pi}\overline{V}_{B}dP$$

assuming solvent is incompressible

 $(\overline{V}_B$ doesn't change with pressure at constant T)

$$\begin{split} \mu_{\mathcal{B}}^{left}\left(X_{\mathcal{B}},P_{0}+\pi\right) &= \mu_{\mathcal{B}}^{left}\left(X_{\mathcal{B}},P_{0}\right) + \left[P_{0}+\pi-P_{0}\right]\overline{V_{\mathcal{B}}} \\ \mu_{\mathcal{B}}^{left}\left(X_{\mathcal{B}},P_{0}+\pi\right) &= \mu_{\mathcal{B}}^{left}\left(X_{\mathcal{B}},P_{0}\right) + \pi\overline{V_{\mathcal{B}}} \end{split}$$

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osmotic pressure (III, alter pressure, continued)

$$\begin{split} \mu_{B}^{left}\left(X_{B},P_{0}+\pi\right) &= \mu_{B}^{left}\left(X_{B},P_{0}\right) + \pi \overline{V}_{B} \\ &= \underbrace{\phantom{\left(X_{B},P_{0}+\pi\right)}}_{B^{left}}\left(X_{B},P_{0}+\pi\right) &= \underbrace{\mu_{B}^{*}\left(P_{0}\right) + RT\ln\left(\gamma_{B}X_{B}\right) + \pi \overline{V}_{B}}_{B} \end{split}$$

want π to restore equilibrium such that $\mu_B^{left}(X_B, P_0 + \pi) = \mu_B^{\bullet right}(P_0)$

$$\begin{split} \underbrace{\mu_{s}^{\star}\left(P_{o}\right) + RT \ln\left(\gamma_{s}X_{s}\right) + \pi\overline{V_{s}}}_{left} &= \underbrace{\mu_{s}^{\star}\left(P_{o}\right)}_{right} \\ \pi &= -\frac{RT \ln\left(\gamma_{s}X_{s}\right)}{\overline{V_{s}}} \end{split}$$



osmotic pressure (a little more manipulation)

$$\begin{split} \pi &= -\frac{RT \ln \left(\gamma_g X_B\right)}{\overline{V}_B} & \text{A=solute} \\ for \ \gamma_g &\approx 1 \quad and \ X_g = 1 - X_A & \text{B=solvent} \\ \pi &= -\frac{RT \ln \left(1 - X_A\right)}{\overline{V}_B} & \text{In}(1 + X_A) \\ \ln (1 + x) &\approx x \quad for \ small \ x \quad (i.e. \ dilute \ solution, \ X_A \ small) \\ \pi &= \frac{X_A RT}{\overline{V}_B} & \\ X_A &= \frac{n_A}{n_A + n_B} & and \ n_A + n_B = n_B \quad for \ dilute \ solution \\ \pi &\approx \frac{n_A RT}{n_B \overline{V}_B} & \\ \pi V_B &= n_A RT & \\ \pi V_{solution} &= n_{solute} RT & \\ \end{split}$$

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colligative properties and nonvolatile solutes

- mole fraction solvent X_B =n_B/n_{total}
- for non-volatile solute A $P_A^{\bullet}=0$
- concentrations frequently given in molality M=molarity=(moles solute)/(Lsolution)
 †m=molality=(moles solute)/(1000g solvent) moles H₂O/1000g solvent=55.55 moles aqueous soln: X_{solvent}=55.55/(m+55.55)
- for ionic solvent (moles solute)=(total moles of ions)
- example: 2 ${\bf m}$ aqueous solution of ${\rm CaCl_2}$ (complete dissociation)

 $X_{H,O} = \frac{55.55 \text{ moles / kgH O}}{(2 \text{moles solute/kgH,O} \times 3 \text{moles ions/moles solute}) + 55.55 \text{moles / kgH,O})} = .9025$

† however when referring to the molality of a specific ion, e.g. [Cl-] from CaCl₂, the number of ions per mole of solute is already factored in